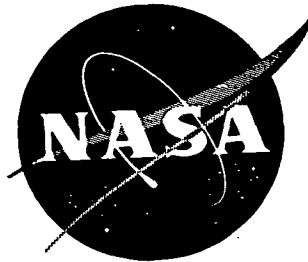


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COMPUTER ANALYSIS OF TWO-DIMENSIONAL
FATIGUE FLAW-GROWTH PROBLEMS

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ABSTRACT

A description is given of a computer program for analyzing the crack propagation in cyclic loaded structures. By using equations for the stress-intensity factors for surface and embedded flaws, the capability of ascertaining lengthwise crack growth as well as crack growth through the thickness of the structure was determined. Equations used are written in general form so that many other types of crack-growth problems, such as through-the-thickness type cracks, can be analyzed. Provisions have been made in the computer program to analyze crack growth for repeated blocks of variable-amplitude fatigue load spectra. Additional features of the computer program include a variable print interval and a variable integration interval to minimize computer printout and computer run time. A source listing and instructions for using the computer program are included.

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COMPUTER ANALYSIS OF TWO-DIMENSIONAL FATIGUE FLAW-GROWTH PROBLEMS

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SUMMARY

A computer program was developed to analyze crack propagation in cyclic loaded structures for both one-dimensional and two-dimensional flaw-growth problems. A sample two-dimensional problem was solved for the growth to crack breakthrough of a surface flaw, and the calculated results were found to compare favorably with experimental results.

This analytic capability was developed in order to help provide the Space Shuttle Program with a reusable vehicle with a long service life. Future work in the area of fatigue crack-growth analysis and testing will consist of deriving a more accurate theoretical solution for the stress-intensity factor of a surface flaw, and studying the flaw-growth rate behavior of different structural materials to be used in the Space Shuttle Program.

INTRODUCTION

In the Apollo Program, fracture mechanics was used as an analytical and empirical tool to implement fracture control and thus prevent catastrophic failure of main propellant tanks, pressure vessels, and other structural components. The service life of these components was quite short and usually limited to a proof test, a few operational pressure tests, and a single mission cycle. The method of fracture mechanics analysis applied to the Apollo structural components is described in reference 1.

Unlike the Apollo Program, the planned Space Shuttle Program will employ a reusable vehicle with a long service life, and consequently, the fatigue environment and crack-growth problems will be considerably more complex. Because of the longer fatigue life and more complex loading, the pressure vessels, main propellant tanks, and other structural components will require improved crack-growth predictions. Not only will the number of load cycles to fracture be important, but also, in the case of thin-walled tanks, the number of cycles to leak must be considered. Accurate crack-growth predictions will be necessary to define test requirements, nondestructive test flaw detection requirements, and appropriate inspection intervals.

To provide a capability for more general and complex flaw-growth analysis relevant to the Space Shuttle Program, a computer program has been developed to perform

analysis for both one-dimensional and two-dimensional flaw-growth problems. The purpose of this report is to describe the theoretical background and the use of this computer program.

As an aid to the reader, where necessary the original units of measure have been converted to the equivalent value in the Système International d'Unités (SI). The SI units are written first, and the original units are written parenthetically thereafter.

SYMBOLS

The SI unit conversion factors used with these symbols are listed in appendix A.

A	crack depth or crack length, mm (in.)
a	semiminor axis of ellipse
B	crack half-width or a geometrical dimension, mm (in.)
b	semimajor axis of ellipse
CA	material constant for crack growth in the A-direction
CB	material constant for crack growth in the B-direction
DP	stress range in a fatigue cycle, maximum P - minimum P, MN/m^2 (ksi)
F	correction factor for the effect of the front surface on the growth of a crack through the thickness
G	correction factor for the effect of the front surface on the growth of a crack in the width direction
K	stress-intensity factor, $\text{MN}/(\text{m})^{3/2}$ ($\text{ksi}\sqrt{\text{in.}}$)
KC, K_{IC}	critical stress-intensity factor for fracture, $\text{MN}/(\text{m})^{3/2}$ ($\text{ksi}\sqrt{\text{in.}}$)
M	correction factor for the effect of the back surface on the growth of a crack through the thickness
N	number of fatigue load cycle
P	applied stress, MN/m^2 (ksi)
P_0	stress level at which crack occurs, MN/m^2 (ksi)
Q	flaw-shape correction factor

R	ratio of minimum applied stress to maximum applied stress in a fatigue cycle
S	numerical exponent in the fatigue crack-growth equation
T	plate thickness, mm (in.)
Y	material yield strength, MN/m ² (ksi)
ΔK_A	stress-intensity factor range used for calculating crack growth in the A-direction, MN/(m) ^{3/2} (ksi $\sqrt{\text{in.}}$)
ΔK_B	stress-intensity factor range used for calculating crack growth in the B-direction, MN/(m) ^{3/2} (ksi $\sqrt{\text{in.}}$)
Φ	complete elliptic integral of the second type
β, θ	angular coordinates

DISCUSSION

Before describing the improved computer analysis developed for fatigue flaw-growth problems, several comments will be made concerning the crack-growth analysis approach used in the Apollo Program. For that analysis, the theory was restricted to elliptically shaped surface and embedded types of flaws and can be expressed by the following functional relationship obtained from reference 1.

$$\frac{d\left(\frac{A}{Q}\right)}{dN} = f\left(\Delta K, \frac{P_o}{P}\right) \quad (1)$$

Two assumptions that are unique and should be noted are made in equation (1). The first assumption is that crack-growth rate can be normalized with respect to the flaw-shape parameter Q . This assumption conflicts with previous crack-growth theories which assume that crack-growth rate at a local point at a crack boundary is governed primarily by the stress-intensity factor at that point (ref. 2). The normalization becomes particularly questionable for large variations in Q such as in the case of a long surface flaw ($A/B < 0.1$) that grows into the shape of a semicircular flaw before fracture.

The second assumption in equation (1) is that the growth rate must be corrected for the stress-level ratio P_o/P . In the ratio, P_o is the stress level at which crack-growth rate data are obtained, and P is the stress level of the crack problem to be analyzed. This ratio was introduced because it was found to be helpful to assume that cycles to fracture N_c was a function only of the ratio K_{II}/K_{IC} . Curves of K_{II}/K_{IC}

compared with N_c could be generated from test data, and the crack-growth rate could be determined at a value of K_{II}/K_{IC} by measuring the slope of the curve at that value.

The validity of correcting for the ratio P_o/P has never been sufficiently investigated, but the correction is certainly questionable for very small and very large values of P_o/P . Applying the correction also conflicts with crack-growth rate theories found to be accurate for through-the-thickness crack-growth problems.

The crack-growth theory used for the computer analysis described in this report was obtained from reference 3. The theory was originally developed for one-dimensional analysis, or growth in one crack dimension only. The theory was investigated in references 4 and 5 and was found to be the most accurate one available for analysis of through-the-thickness cracks in aluminum sheet material. However, the theory has been extended to account for two-dimensional flaw growth, or change in flaw size in two principal directions for surface and embedded types of flaws. The approach developed is to not normalize flaw-growth rate as is done in equation (1), but to assume that flaw-growth rate in the two principal directions is a function of the stress-intensity factors in these directions.

Other features of the computer program include the ability to analyze numerous problems consecutively, to analyze variable amplitude fatigue loading with different values of the load ratio R and to have block loading that is relevant to studying the effect of repeated Space Shuttle missions.

In the following sections, a complete mathematical formulation of the analysis is described, and instructions for use of the program are provided. Also, comparisons are shown between calculated and experimental results. The program is written in FORTRAN V for the Univac 1108 computer. Conversion factors for the International System of Units (SI) are noted in appendix A. A source listing of the program is given in appendix B, and the input to a sample problem along with the resulting printout is given in appendix C.

Mathematical Formulation

The basic equations to be solved by the computer program are the differential equations for crack-growth rates in two principal directions (e. g., through the depth of the material and in the plane of the material). The equations to be solved using the crack-growth rate theory from reference 3 have the following forms:

$$\frac{dA}{dN} = \frac{CA(\Delta K_A)^S}{(1-R)KC - \Delta K_A} \quad (2)$$

$$\frac{dB}{dN} = \frac{CB(\Delta K_B)^S}{(1-R)KC - \Delta K_B} \quad (3)$$

The crack-growth rate theory in reference 3 is the most accurate and the best available theory for through-the-thickness cracks. Comparisons with experimental data show that the theory is also valid for two-dimensional flaw-growth analysis.

The greater part of the work in the computer analysis is to determine the values of the parameters ΔK_A and ΔK_B and then to solve differential equations (2) and (3) by using the Runge-Kutta numerical integration method. For determining ΔK_A and ΔK_B , Irwin (ref. 6) has derived the following generally accepted equation for the stress-intensity factor at any location on the crack border of an elliptical crack in an infinite-thickness solid. (See fig. 1(a).)

$$K = P \left(\pi \frac{a}{Q} \right)^{1/2} \left(\sin^2 \beta + \frac{a^2}{b^2} \cos^2 \beta \right)^{1/4} \quad (4)$$

where

$$Q = \Phi^2 - 0.212 \left(\frac{P}{Y} \right)^2 \quad (5)$$

and

$$\Phi = \int_0^{\pi/2} \left[1 - \left(\frac{b^2 - a^2}{b^2} \right) \sin^2 \theta \right]^{1/2} d\theta \quad (6)$$

By substituting symbols and appropriate values of β as shown in figures 1(b) and 1(c), the following relationships for ΔK are obtained for the elliptical or semielliptical crack in a finite thickness solid. (See fig. 2.) For a crack propagating in the thickness direction

$$\Delta K_A = F(DP)M \sqrt{\frac{\pi A}{Q}} \quad (\text{if } A \leq B) \quad (7)$$

$$\Delta K_A = F(DP) \left(\frac{B}{A} \right) \sqrt{\frac{\pi A}{Q}} \quad (\text{if } A > B) \quad (8)$$

For a crack propagating in the width direction

$$\Delta K_B = G(DP) \left(\frac{A}{B} \right) \sqrt{\frac{\pi B}{Q}} \quad (\text{if } A \leq B) \quad (9)$$

$$\Delta K_B = G(DP) \sqrt{\frac{\pi B}{Q}} \quad (\text{if } A > B) \quad (10)$$

In equations (7) to (10), the usual procedure in fracture mechanics of including correction factors for the finite thickness of a plate (i. e., factors F, G, and M) has been followed. By substituting equations (7) to (10) into equations (2) and (3), all parameters are now a function of the crack geometry and the working stresses and are readily available for the solution of the problem with the exception of F, G, M, and Q.

The parameter F is a correction factor for the effect of the front surface on growth through the thickness. This correction factor has been assumed by Kobayashi and Moss (ref. 7) to be given by the equation

$$F = 1.0 + 0.12 \left(1 - \frac{A}{2B} \right)^2 \quad (11)$$

Equation (11) is a proposed interpolation between two extreme cases, Smith's theoretical solution for $A/2B = 0.5$ (ref. 8) and Bowie's theoretical solution for $A/2B = 0$ (ref. 9). Since the extreme values of F vary only between 1.03 and 1.12, the proposed interpolation should give acceptable accuracy.

To make the computer analysis general enough so that problems other than the elliptical flaw can be studied (e. g., one-dimensional through-crack problems), the value of F can be put in either as a constant or as a table. The values for F determined from equation (11) are listed in table I. For other problems, or for different correction factors other than those given by equation (11), the values need only to be substituted for those in table I. If a one-dimensional through-crack problem is to be analyzed, the proper values for the correction factors must be substituted into equation (7), and the option selected in the program to integrate equation (2) only. (Equations (8), (9), and (10) are ignored.)

The parameter G is a correction factor for the effect of the front surface on growth in the width direction. No solution for G has been derived for the semiellipse, but Thresher and Smith (ref. 10) have solved the problem of a partially embedded circular crack by numerical methods. By assuming equal depths and surface lengths for circular and semielliptical surface cracks, the correction factor G can be estimated for the semielliptical crack. This analysis was performed, and some values for G are listed in table II. Because almost all values are approximately 1.12, the procedure shown in figure 1 is to let G be a constant with a value of 1.12.

The parameter M is a correction factor for the effect of the back surface on growth through the thickness. Several approximate solutions exist for M . (See refs. 7 and 11.) For this computer program, the input for the parameter can be either a constant or a table. The values obtained from a recent solution by Shah and Kobayashi are listed in table III and are used in the program. This solution was selected because it is the latest and most accurate numerical solution available. To reduce the complexity of the input requirements, the table for M was programed as data statements into the main program. Details for making changes for this table will be given in the section entitled "Users' Input Instructions."

The final parameter, Q , is a modification of equation (5) to account for cyclic loading at different load ratios. The modified equation for Q is

$$Q = \Phi^2 - 0.212 \left[\frac{DP/(1-R)}{Y} \right]^2 \quad (12)$$

where Φ is the elliptic integral given by equation (6). This elliptic integral is a function of the crack geometry and is also put into the program either as a constant value or with the values listed in table IV. The accuracy of equation (12) is difficult to determine, but it should be approximately as accurate as equation (5). Equation (12) assumes small-scale yielding at the crack tip and plane strain conditions. This assumption would be less accurate for relatively high-toughness materials and for calculating ΔK_B of a surface flaw where plane stress conditions may exist.

It can be seen that the solution of equations (2) and (3) is dependent only upon the availability of the initial crack geometry, the material constants, and the applied cyclic stresses. Growth of the crack for each cycle of load will then be calculated by the computer.

Description of the Program

Generally, the program integrates the crack propagation rate, equations (2) and (3), to obtain the crack growth in a structure for both the crack depth and the crack half-width at each cycle of load. These load cycles can have variable amplitudes and can be repeated for as many times as the user wishes in order to obtain the complete growth history for the life of the structure.

The program consists of a main program and six subroutines. The main program is called CRACK and has the responsibility of reading in all data, shifting control to the subroutine called CRANE for performing the calculation, and then writing out the results.

Subroutine CRANE does the actual integration of equations (2) and (3) and is dependent on a subroutine called DERIV to supply the derivatives. After the integration has been completed, the control is shifted back to the main program CRACK to write the results.

Subroutine DERIV sets up the derivatives needed for CRANE. The derivatives calculated (depending on the type of flaw) are dA/dN and dB/dN as given by equations (2) and (3). The only derivative that is dependent upon a value of M is equation (2). The parameter M has been inserted as a part of the program and is expressed as either a constant, having a value of 1.0, or in tabular form for a two-dimensional curve of A/T and A/B compared with M as shown in table III. If the tabular values are required for the solution of the problem as in figure 2, then subroutine DISCOT will perform the interpolation for values of M between those given in the table and supply the correct value for the calculation in DERIV. Both equations (2) and (3) are dependent upon a value of Q which is a constant for the case of one-dimensional flow growth and is a variable for two-dimensional flow growth. If a two-dimensional flow-growth problem is to be analyzed, then the user must supply a table of A/B or B/A versus Φ^2 such as the one shown in table IV. Subroutine DISCOT will perform the interpolation for values of Φ^2 for the crack geometry and supply the correct value for the calculation of Q and the subsequent calculations of equations (7) to (10). This same procedure holds for evaluation of the parameter F if it were given as a table.

Subroutine DISCOT is an interpolation routine for determining intermediate points on a curve such as those that would be in the tabular form given above. This subroutine is responsible for determining the proper M , Q , or F values to be used in the calculations of subroutine DERIV. Subroutine DISCOT is dependent upon three other subroutines called UNS, DISSER, and LAGRAN to perform this interpolation.

The general flow of the problem is given as follows. Depending on the type of flaw to be analyzed, the user supplies the original crack geometry, the required material constants, the working stress levels, and the other parameters that are needed for program control. After these cards have been read by the computer, the control is transferred to subroutine CRANE which sets up the necessary format for performing the integration. CRANE then transfers control to subroutine DERIV for calculation of the derivatives.

DERIV is common with the main program and already contains the input crack data and other parameters needed for the solution. Solution of equation (2) also requires a value for the magnification factor M which has been inserted into the program as both a constant ($M = 1.0$) and as a table. When a tabular value is required, DISCOT is called to determine M by first calculating the A/T and A/B ratios and then interpolating the table for the corresponding value of M . This procedure is repeated in the same manner to obtain values of Q and F . With these values, DERIV calculates the derivatives dA/dN or dB/dN (or both) and transfers these values back to CRANE.

CRANE performs a stepwise integration of the derivatives to determine the changes in the crack parameters A and B , then adds them to the original values. These changed values are then transferred back to the main program and printed out. They are also carried back to subroutine DERIV instead of the original values, and the complete process is repeated until one of two events happens. First, for surface or embedded cracks (fig. 2), if the crack depth becomes equal to or greater than the thickness of the material, the run is terminated. Second, for any type of crack, if the denominator of equation (2) or (3) becomes negative or equal to zero, the integration will

become unstable, and the run will be terminated. If neither of these events happens, then the run will continue normally until the final cycle of the last load case has been reached.

Users' Input Instructions

The first card of the data deck is a title card for the description of the problem. The other cards contain information such as material properties, geometry definition, loading, controls on print and integration interval, and repetition of load spectrum. Detailed instructions for punching data on the cards for each of the flaw types are given below. The order of the data deck is shown in figure 3.

Two-dimensional flaw-growth problems. - The two-dimensional flaw-growth problems are solved by the use of nine cards which are described as follows:

1. Card 1 (one card to a problem)

This card is a title card for the case being run and may contain up to and including 78 columns of alphanumeric information.

2. Card 2 (one card to a problem)

Column no.	Parameter	Format
1 to 10	Flaw-growth type number	(F10.0)

NOTE: If the analysis is for a one-dimensional flaw-growth problem, then 1.0 goes in this field. If the analysis is for a two-dimensional flaw-growth problem, then 2.0 goes in this field.

3. Card 3 (one card to a problem)

Column no.	Parameter	Format
1 to 10	AI	(F10.0)
11 to 20	T	(F10.0)
21 to 30	B	(F10.0)

NOTE: Initial flaw dimensions are AI and B.

4. Card 4 (one card to a problem)

Column no.	Parameter	Format
1 to 10	KC	(F10.0)
11 to 20	Y	(F10.0)
21 to 30	S	(F10.0)
31 to 40	G	(F10.0)
41 to 50	CA	(E10.0)
51 to 60	CB	(E10.0)

NOTE: The values for CA and CB are usually very small and should be punched in the following manner. If the value for CA is given as 0.5×10^{-13} , then it is input as CA = 0.5 E-13. The E-13 is "right justified" and must end in column 50. The 0.5 may be punched anywhere (with decimal) between and including columns 41 and 46.

5. Card 5 (one card to a problem)

Column no.	Parameter	Format
1 to 10	NI	(F10.0)
11 to 20	DN	(F10.0)
21 to 30	START	(F10.0)
31 to 40	NL	(I10)
41 to 50	NBLK	(I10)

NOTE: The initial cycle number is NI. The desired cycle print interval is DN. START is used to omit the printout up to a cycle number put in for this parameter; then after the START cycle, the printout will be in accordance with the number put in for DN. If the value for NF is put in for START, only AI and the final cycle will be printed. Counters for the program are NL and NBLK. The value given to NL tells the computer when it has read the last of the loading cards (card 9); therefore, the number of 9 cards and the value for NL must agree. The value given to NBLK tells the computer to repeat the number of NL load conditions an NBLK number of times.

6. Card 6 (one card to a problem)

Column no.	Parameter	Format
1 to 10	IQ	(I10)
11 to 20	MQTBL	(I10)
21 to 30	IF	(I10)
31 to 40	MFTBL	(I10)

NOTES: (a) The values for both IQ and IF are integers and are used as counters to tell the computer when it has read the last of cards 7 and 8, respectively. Therefore, these values must agree with the number of cards 7 and 8 for the problem. The maximum value of IQ and IF is 100 which means that the user can input a maximum of 100 cards to describe the tables for Φ^2 and F. For a larger number, the program must be changed internally.

(b) MQTBL and MFTBL are also integers and used as flags in the program that eliminate the need for repunching the table of cards 7 and 8 for every problem that uses these identical tables. For example, if several problems require the same Φ^2 and F tables, then these tables will only have to be punched once and presented to the computer in the first problem using the tables. For the first problem or a single problem, the values for MQTBL and MFTBL should be 1 or any value greater than 1. For subsequent problems using these same Φ^2 and F tables, the values of MQTBL and MFTBL will be 0 or left blank.

7. Card 7 (IQ number of these except when MQTBL = 0 or blank)

Column no.	Parameter	Format
1 to 10	AB(I)	(F10.0)
11 to 20	QQ(I)	(F10.0)

NOTE: If IQ = 1 from card 6, then Q is a constant and the value of Q (not Φ^2) is punched in columns 11 to 20, and the value of AB(I) is omitted. There will only be one card 7 in this case.

8. Card 8 (IF number of these except when MFTBL = 0 or blank)

Column no.	Parameter	Format
1 to 10	AAB(I)	(F10.0)
11 to 20	F1(I)	(F10.0)

NOTE: The note of card 7 also pertains to this card except replace comments for AB(I) with AAB(I) and QQ(I) with F1(I).

9. Card 9 (NL number of these)

Column no.	Parameter	Format
1 to 10	NF(I)	(F10.0)
11 to 20	DP(I)	(F10.0)
21 to 30	R(I)	(F10.0)
31 to 40	HH(I)	(F10.0)
41 to 50	PRC(I)	(F10.0)
51 to 60	ISC(I)	(I10)

NOTES: (a) The initial step size for subroutine CRANE is HH (printed out as H). HH must not equal zero but may be any other value the user chooses. This is a stepwise increment for the integration routine and is usually set to 1.0 or greater. (By letting HH be equal to a multiple of 10.0 where $HH < (NF - NI)/1000$ will usually give satisfactory results with a minimum of computer time.) The step size will be held constant in the program as long as $ISC = 1$. If the user sets $ISC = 0$, then the step size will be changed internally when the integration starts to become unstable.

(b) Subroutine CRANE uses precision parameter PRC. This is a positive constant used in the integration routine that is approximately equal to the number of significant decimal digits of local precision. The truncation error comes into play only when the step size flag $ISC = 0$.

(c) The parameter ISC is an integer and has a value of 1 or 0. If $ISC = 1$, then the step size (internally) will be held constant through all calculations. If $ISC = 0$, then the step size will be allowed to vary and become smaller than H as the integration routine starts to become unstable.

One-dimensional flow-growth problems. - Two typical one-dimensional flow-growth problems are shown in figure 4. The values of the parameters F, Q, and M for these problems, along with the values for many other problems, can be obtained from reference 12.

With the exceptions of cards 3 and 4, the formats for one-dimensional flow-growth problems 1 to 9 (fig. 4) are the same as they are for two-dimensional flow-growth problems. Some input values, however, are different. Since Q will be a constant, IQ is punched as 1 on card 6, and the constant value of Q (e.g., equal to 1.0) should be punched in columns 11 to 20 on card 7. The value of AB(I) should be left blank on card 7 when Q is a constant.

The input values for the factor F depend entirely upon the problem to be solved or the values the program user desires to use (ref. 4). If the value of F is to be a constant, the same procedure described for letting Q be a constant should be used.

Cards 3 and 4 have different formats than those used for two-dimensional flow-growth problems. The required formats are as follows.

1. Card 3 (one card to a problem)

Column no.	Parameter	Format
1 to 10	AI	(F10.0)
11 to 20	B	(F10.0)

2. Card 4 (one card to a problem)

Column no.	Parameter	Format
1 to 10	KC	(F10.0)
11 to 20	S	(F10.0)
21 to 30	CA	(E10.0)

Values of M for Varying A/T and A/B

The input for the table containing values of M for varying A/T and A/B is somewhat complex, as can be seen from table III. To eliminate the need to input this table for every problem that uses it, the table has been inserted into the main body of the program deck and is called on when required. Those data cards can be seen as cards CRAC 0450 to CRAC 0570 in the main program deck CRACK.

If there is reason to change this table, the user should perform the following steps:

1. Make a new table III or make the necessary changes to the old one.
2. Notice that the A/B ratios in table III and on card CRAC 0450 are increasing; for example: $A/B = 0$ to $A/B = 1.0$. (Assumed upper bound values are determined from reference 13.) Similarly, the A/T ratios of table III and card CRAC 0460 are also increasing ($A/T = 0$ to $A/T = 1.0$). The table should be input in this manner (both ratios increasing) for use in the interpolation routine of DISCOT.
3. The data for M are then input column by column in the increasing ratio direction. For example, from table III, the first column of M 's (for card CRAC 0470) is for the ratio of $A/T = 0$ and is input from $A/B = 0$ to $A/B = 1.0$. The second column (for card CRAC 0480) is for $A/T = 0.1$ and is input from $A/B = 0$ to $A/B = 1.0$. This procedure is repeated for the remaining columns of M .

For this particular table, there is enough room on each card to punch one column of M 's per card. It should also be noted that this particular table has 11 rows and 11 columns, thus there will be 121 values of M . If the new table has more values or less values of M , then card CRAC 0440 will have to be changed to agree with this number where $IMK = \text{number of values of } M \text{ in the table}$.

It should also be noted that the program has been set up for a maximum number of 200 values for M and 100 values each of ratios A/T and A/B . If more values are required to describe the table, then the appropriate changes should be made to the dimension statements of cards CRAC 0030 and CRAC 0040.

Sample Problem

To provide an example of the use and accuracy of the computer program, a sample problem is furnished along with experimental results for comparison. The sample problem selected is the calculation of fatigue cycles to crack breakthrough of surface-flawed 2219-T87 aluminum alloy plates. This problem is presented because accurate experimental data are available for determining the parameters CA , CB , KC , and S and thus allow a valid comparison with the analytically derived results. Some of this data are shown in figure 5. The data were obtained by putting initial surface cracks in specimens and fatigue loading them at predetermined stress levels and numbers of cycles to obtain small increments of crack growth. The initial and final crack dimensions were determined by pulling the specimens to failure and measuring the dimensions with an optical measuring microscope.

Additional tests were conducted on four specimens to determine the number of cycles required for crack breakthrough to the back face of the specimen. Crack breakthrough was determined by clamping a small chamber filled with water at low pressure on the front side of the specimen and noting when water leaked through to the backside. The results of these tests are presented in table V.

The test results in table V were compared with calculated results from the computer program. The input for the problem and the printed results are given in appendix C. The comparison of experimental and calculated results are shown in figure 6. The comparison is satisfactory, except that the computer results are slightly conservative for all four specimens. The comparison could be improved by adjusting the parameter CA, but figure 5 indicates that this parameter has approximately the correct value. Another cause of the error could be the inaccuracy of the flaw depth correction factor M when applied to thin aluminum specimens. More theoretical and experimental research is required, however, to resolve this difficulty.

The sample problem also assumes that the constant CB is equal to 2CA. This value for CB gives a satisfactory comparison for flaw growth in the width direction. The difference in values between CA and CB is required because the flaw growth on the surface is at a different state of stress than for the growth through the thickness. When analyzing embedded flaws, the state of stress would be the same everywhere at the crack front, and the flaw-growth constants CA and CB should have equal values.

Finally, the sample problem is for a single load level case, or uniform-type fatigue loading. If multiple-load level fatigue is to be analyzed, only cards 5 and 9 are affected. Input instructions for a typical three-load level problem is shown in figure 7.

CONCLUDING REMARKS

The results of the effort to develop an improved computer analysis for use in fatigue flaw-growth problems can be summarized as follows:

1. A theoretical approach originally developed for one-dimensional or through-the-thickness type cracks has been extended to the analysis of two-dimensional fatigue flaw-growth problems.
2. Using the improved approach, a computer program was developed for predicting fatigue flaw growth of surface, embedded, and through-the-thickness cracks with variable-amplitude fatigue load spectra.
3. The theoretical results compare favorably with test results from surface-flawed specimens of 2219-T87 aluminum alloy.
4. The accuracy of the computer analysis could be improved if less approximate theoretical solutions for the stress-intensity factors of embedded and surface cracks were available.
5. Necessary follow-on work planned includes crack-growth testing and analysis of different structural materials, testing and analysis of complex structural configurations such as integrally stiffened panels, and derivation of a more accurate theoretical solution for the stress-intensity factor of a surface-type flaw.

Manned Spacecraft Center

National Aeronautics and Space Administration

Houston, Texas, February 11, 1972

908-42-38-00-72

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TABLE I. - CORRECTION FACTOR F FOR
EFFECT OF FRONT SURFACE ON
GROWTH THROUGH THE DEPTH

<u>A/B</u>	<u>F</u>
0	1.120
0.1	1.108
.2	1.097
.3	1.087
.4	1.077
.5	1.067
.6	1.059
.7	1.051
.8	1.043
.9	1.036
1.0	1.030
2.0	1.000

TABLE II. - CORRECTION FACTOR G FOR
EFFECT OF FRONT SURFACE ON
GROWTH IN WIDTH DIRECTION

<u>A/B</u>	<u>G</u>
0.4200	1.127
.5000	1.121
.5774	1.120
.6546	1.122
.7338	1.108
.8164	1.094
1.0000	1.250

TABLE III. - VALUES OF M FOR VARYING A/T AND A/B^a

$\frac{A/T}{A/B}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0 ^b
0 ^c	1.000	1.005	1.021	1.048	1.091	1.155	1.250	1.400	1.667	2.294	3.100
0.1	1.000	1.003	1.010	1.024	1.042	1.074	1.119	1.193	1.349	1.746	2.350
.2	1.000	1.002	1.007	1.018	1.034	1.065	1.107	1.167	1.279	1.536	1.915
.3	1.000	1.001	1.005	1.013	1.025	1.052	1.090	1.146	1.245	1.430	1.700
.4	1.000	1.000	1.003	1.008	1.018	1.040	1.077	1.131	1.217	1.361	1.580
.5	1.000	1.000	1.001	1.006	1.014	1.030	1.065	1.110	1.190	1.320	1.530
.6	1.000	1.000	1.000	1.005	1.010	1.025	1.051	1.093	1.168	1.291	1.490
.7	1.000	1.000	1.000	1.004	1.008	1.019	1.043	1.082	1.148	1.265	1.455
.8	1.000	1.000	1.000	1.003	1.006	1.013	1.035	1.072	1.131	1.241	1.420
.9	1.000	1.000	1.000	1.002	1.004	1.010	1.029	1.062	1.117	1.220	1.390
1.0	1.000	1.000	1.000	1.001	1.003	1.008	1.024	1.052	1.102	1.199	1.360

^aValues determined from Shah's and Kobayashi's solution.^bValues determined by extrapolating Shah's and Kobayashi's solution.^cAssumed upper bound values determined from reference 13 solution where $M = 1/(1-A^2/B^2)^{1/2}$.

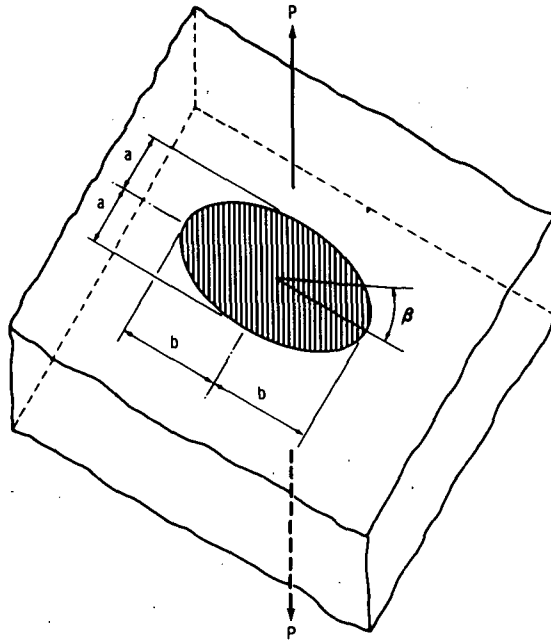
TABLE IV. - VALUES OF A/B OR B/A COMPARED WITH Φ^2

A/B or B/A	Φ^2
0.00000	1.000000
0.22361	1.124605
.31622	1.220527
.38729	1.307354
.44721	1.388838
.50000	1.466656
.54772	1.541746
.59161	1.614772
.63245	1.685915
.67082	1.755688
.70710	1.824239
.74162	1.891730
.77459	1.958297
.80622	2.024049
.83666	2.089074
.86602	2.153444
.89443	2.217225
.92195	2.280468
.94868	2.343220
.97468	2.405517
1.00000	2.467400

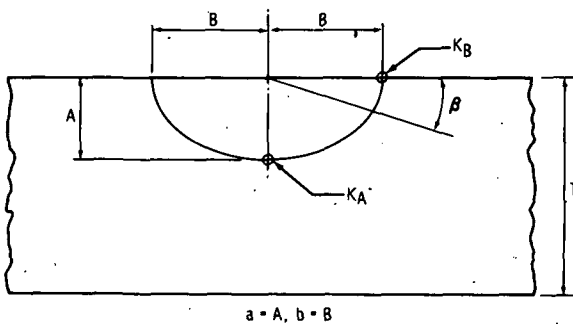
TABLE V. - RESULTS OF CRACK BREAKTHROUGH TESTS OF SURFACE-FLAWED 2219-T87 ALUMINUM

PLATES FATIGUE LOADED IN 70° F DISTILLED WATER

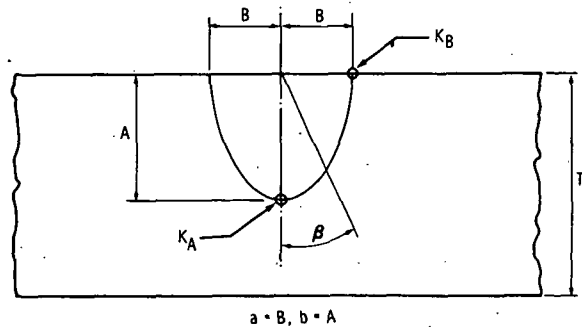
Specimen number	T			Initial flaw size				Final flaw width, B _f		Fatigue loading			Cycles to crack breakthrough
				A _i		B _i				R			
	mm	in.	mm					in.	mm			ksi	
				(MN/m ²)									
9L	3.198	0.1259	1.68	0.066	3.150	0.1240	4.635	0.1825	134.9	19.57	0.01	18 065	
10L	3.162	.1245	1.70	.067	3.213	.1265	4.508	.1775	190.9	27.69	.01	5 272	
12M	3.187	.1255	1.65	.065	3.340	.1315	4.635	.1825	217.5	31.54	.01	3 770	
14M	3.030	.1193	1.60	.063	3.264	.1285	4.648	.1830	285.8	41.45	.01	1 420	



(a) Elliptical crack in an infinite body.

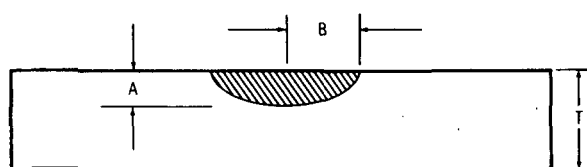
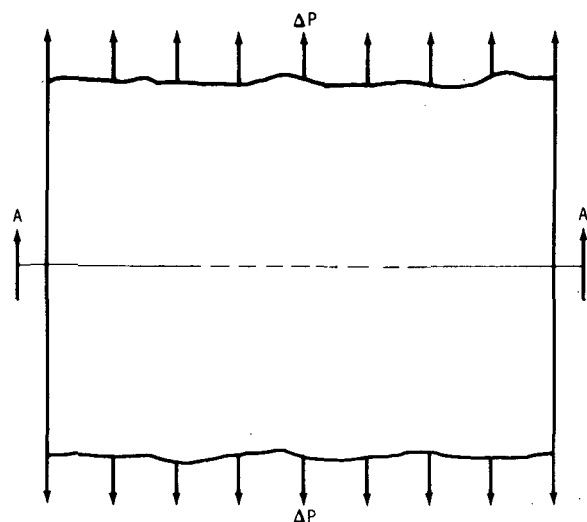


(b) Semielliptical crack where $A \leq B$.

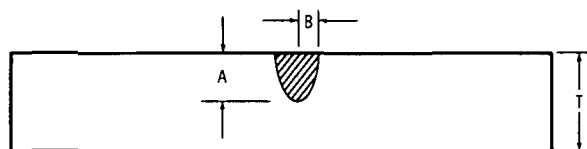


(c) Semielliptical crack where $A > B$.

Figure 1. - Assumed geometry for an elliptical crack in an infinite-thickness body and a semielliptical crack in a finite-thickness body.

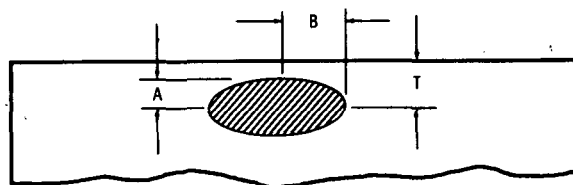
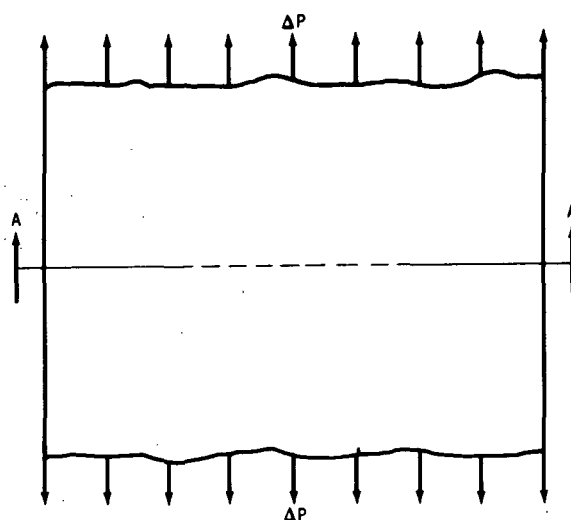


Sec. A-A for $A \leq B$
 $F = f(A/B)$ from table I, $G = 1.12$
 $M = f(A/B, A/T)$ from table III.

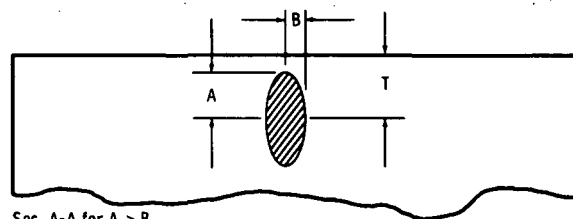


Sec. A-A for $A > B$
 $F = f(A/B)$ from table I, $G = 1.12$
 $M = 1$

(a) Surface flaw.



Sec. A-A for $A \leq B$
 $F = 1, G = 1$
 $M = f(A/B, A/T)$ from table III.



Sec. A-A for $A > B$
 $F = 1, G = 1$
 $M = 1$

(b) Embedded flaw.

Figure 2. - Crack geometry for surface and embedded flaws.

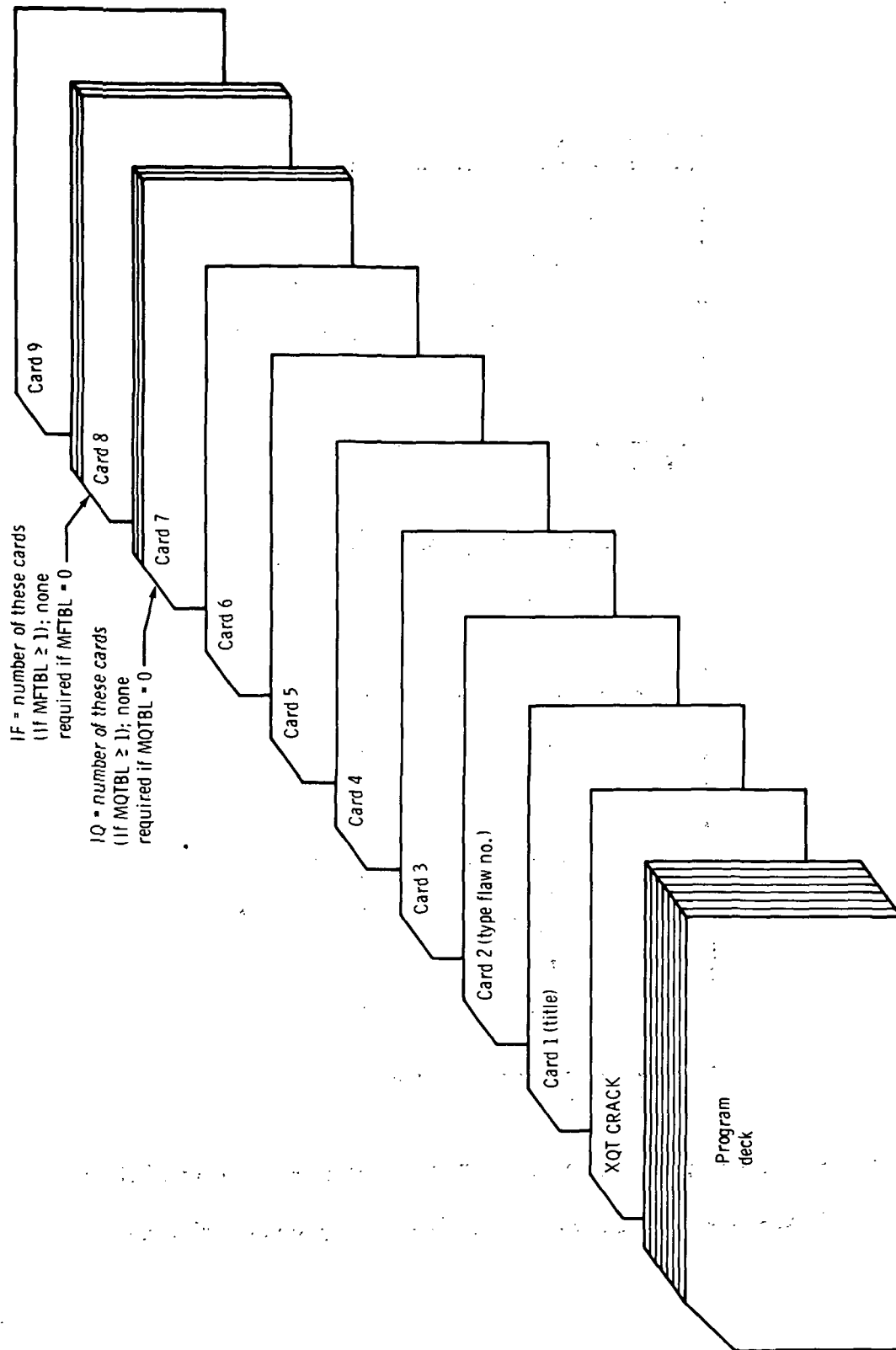
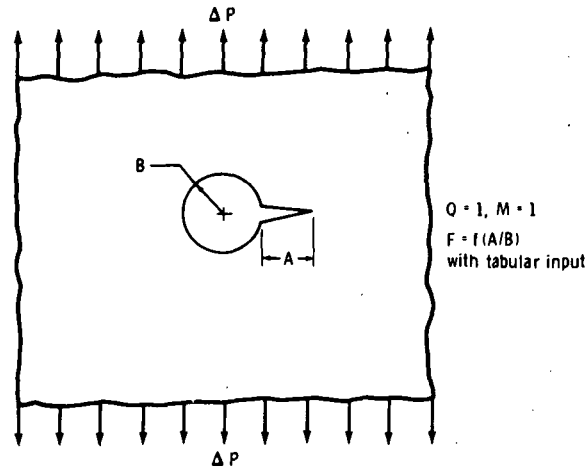
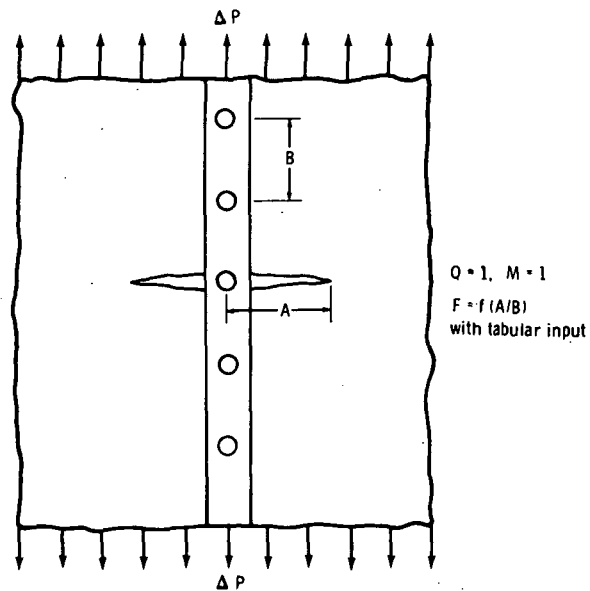


Figure 3. - Order of program and data deck.



(a) Crack propagating from hole.



(b) Symmetric crack propagating from rivet hole in stiffened plate.

Figure 4. - Typical one-dimensional flaw-growth problems.

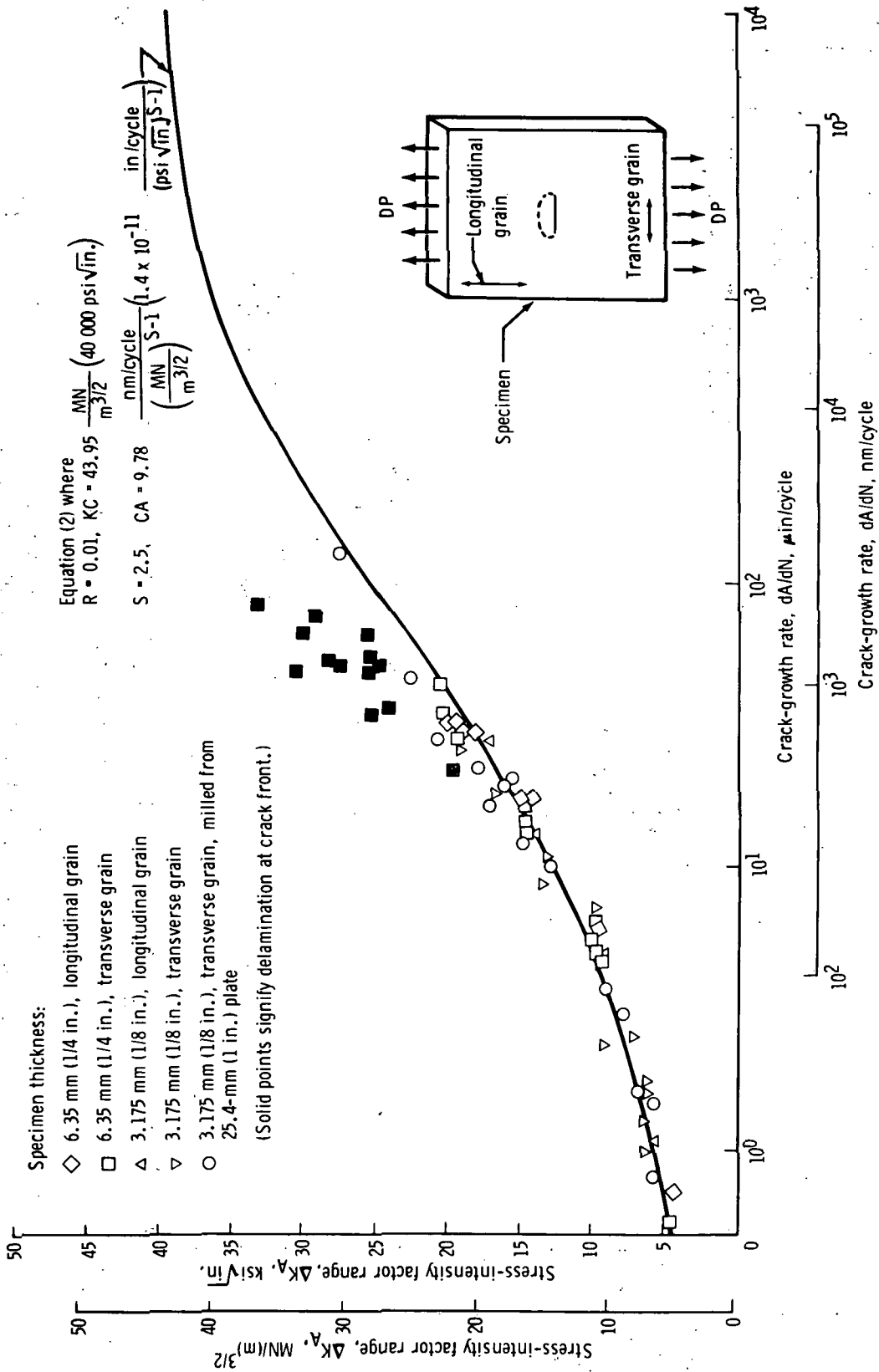


Figure 5. - Comparison of theoretical crack-growth rate with experimental room temperature data for 2219-T87 aluminum plate material.

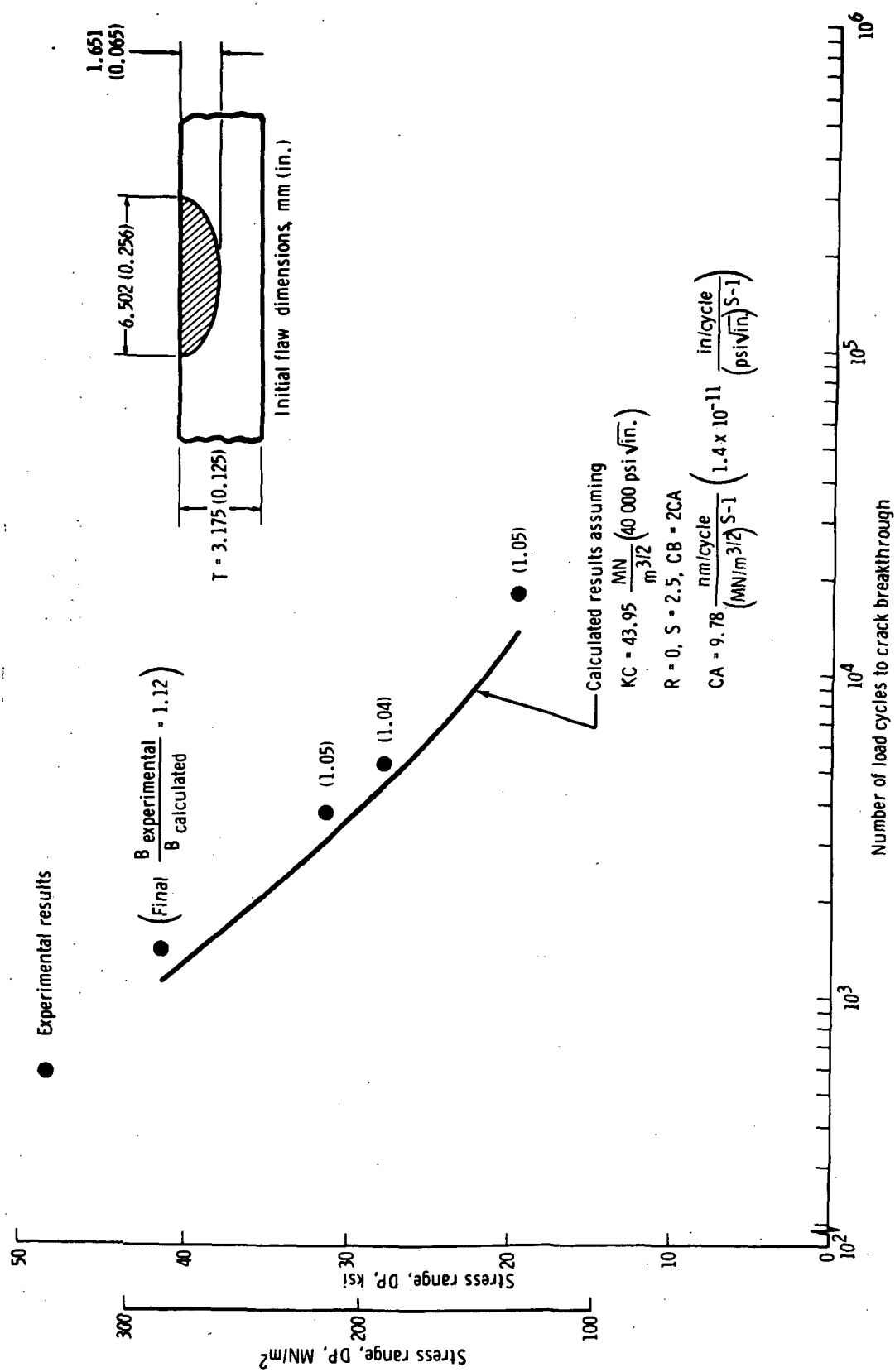


Figure 6. - Comparison of experimental results with calculated results for load cycles to crack breakthrough in 2219-T87 aluminum alloy sheet.

INPUT INSTRUCTIONS

CARD 5:

NI = 0.0
 DN = 100.
 START = 0.0
 NL = 3
 NBLK = 1

CARD 9 (1):

NF = 2000. (cycles)
 DP = 20000. (psi)
 R = 0.0
 HH = 1.0
 PRC = 7.0
 ISC = 1

CARD 9 (2):

NF = 4000.
 DP = 10000.
 R = .666
 HH = 1.0
 PRC = 7.0
 ISC = 1

CARD 9 (3):

NF = 5500.
 DP = 10000.
 R = .5
 HH = 1.0
 PRC = 7.0
 ISC = 1

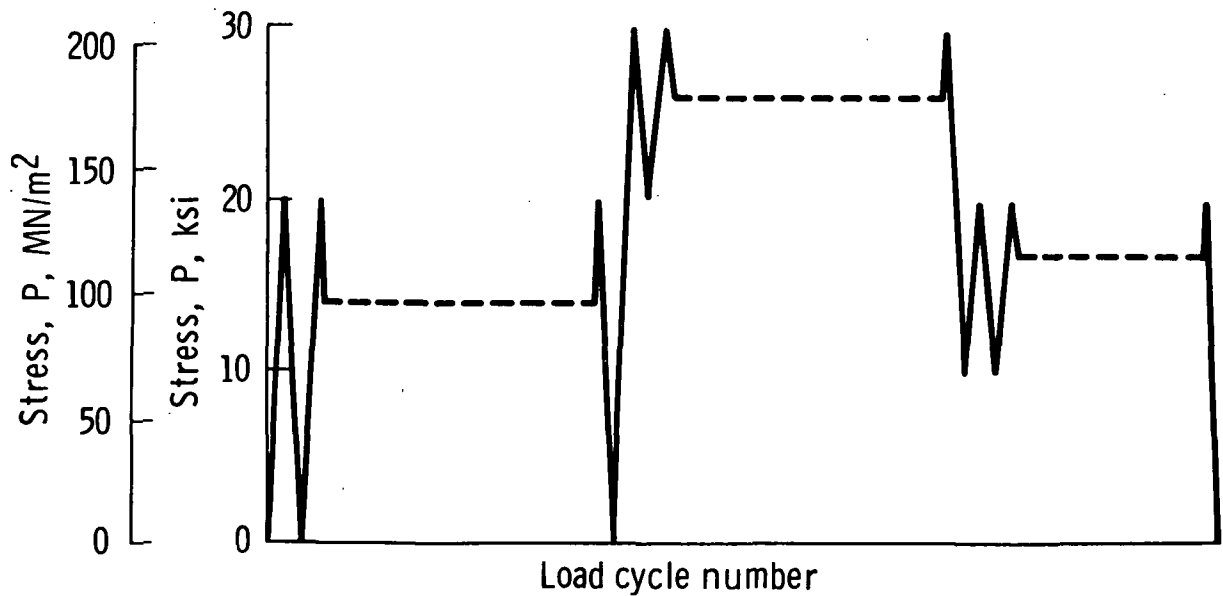
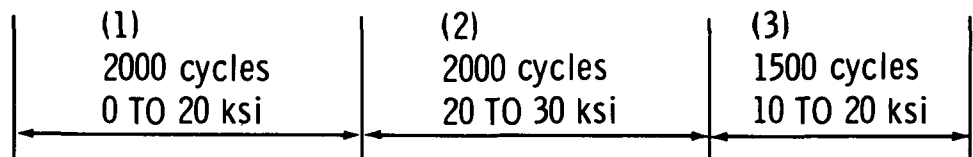


Figure 7. - Input instructions for a typical multiple-load level program.

APPENDIX A

CONVERSION OF U. S. CUSTOMARY UNITS TO SI UNITS

The SI was adopted by the Eleventh General Conference of Weights and Measures, Paris, October 1960, in Resolution Number 12 (ref. 14).

TABLE A-I. - CONVERSION FACTORS FOR SI UNITS

To convert from U. S. Customary Units	Multiply by —	To obtain SI units
lbf	4.448222	newtons (N)
in.	2.54×10^{-2}	meters (m)
kips per square inch (ksi)	6.894757×10^6	newtons/meter ² (N/m ²)
ksi $\sqrt{\text{in.}}$	1.0988	MN/(m) ^{3/2}
$\mu\text{in/cycle}$	25.4	nm/cycle

TABLE A-II. - PREFIXES AND SYMBOLS TO INDICATE
MULTIPLES OF UNITS

Multiple	Prefix	Symbol
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^6	mega	M

APPENDIX B
COMPUTER PROGRAM LISTING

```

10 FOR CRACK
  DIMENSION IO, TITLE(13)
  DIMENSION H(100), PRC(100), ISC(100)
  DIMENSION MMK(200), AR(100), QO(100), AT(100), P(100), DP(100),
  1 NF(100), AM(100), F1(100), AAR(100)
  REAL RF, I, KC, MMK, M
  COMMON /CRANECK/ XIN, A(40), FF(40), H, N, B(40), MIN, PRC, IO, NT, INIT
  COMMON /CRACK/ K, B, I, IMK, IO, AB, AT, MMK, QO, DP, R, S, KC, CA, G, DENOM, I
  1 Y, A1, DENOM2, BK1, BK2, G, ADT, ADR, M, AR, F1, IF, AAR, TYPE, F, CR
  500 READ(5, 9969) TITLE
  FF(1) = 0.0
  FF(2) = 0.0
  6969 FORMAT(13A6)
  WRITE(6, 6970) TITLE
  6970 FORMAT(1H1, 21X13A6//)
  READ(5, 3) TYPE
  3 FORMAT(F10.0)
  IF(TYPE.LT.2.0) GO TO 7
  READ(5, 1) AI, I, B, KC, Y, S, G, CA, CR, NI, DN, START, NL, NPLK, IO, MOTRL,
  1 IF, METRL
  1 FORMAT(3F10.0/4F10.0/2E10.0/3F10.0/2I10/4I10)
  WRITE(6, 2) AI, I, B, KC, Y, S, G, CA, CR, NI, DN, NL, NPLK, IO, IF
  2 FORMAT(45X'CRACK PROPAGATION STUDY'///55X'INPUT'///30X'INITIAL CRACK
  1K DEPTH', 11X, AI = 'E20.8/30X'PLATE THICKNESS', 15X, I = 'E20.8/30X'INCRACK
  2INITIAL CRACK WIDTH', 11X, B = 'E20.8/30X'FRACTURE TOUGHNESS', 12X, KC = 'E20.8/30X'
  3E20.8/30X'MATERIAL YIELD STRESS', 9X, Y = 'E20.8/30X'EXPONENT OF DKCRACK
  4, 10X, S = 'E20.8/30X'CORRECTION FACTOR', 1X, G = 'E20.8/30X'WATERIACRACK
  5L CONSTANT FOR A', 7X, CA = 'E20.8/30X'MATERIAL CONSTANT FOR B',
  67X, CR = 'E20.8/30X'INITIAL CYCLE NUMBER', 10X, NT = 'E20.8/
  730X'CYCLE PRINT INTERVAL', 10X, DN = 'E20.8/30X'NUMBER LOAD CONDITIONCRACK
  8S, 8X, NL = 'E20.8/30X'NUMBER LOAD BLOCKS', 12X, LDS = 'E20.8/
  930X'NUMBER OF G VALUES', 12X, IO = 'E20.8/
  130X'NUMBER OF F VALUES', 12X, IF = 'E20.8/
  60 TO 94
  7 READ(5, 5) AI, B, KC, S, CA, NI, DN, START, NL, NPLK, IO, MOTRL, IF, METRL
  5 FORMAT(2F10.0/2F10.0/E10.0/3F10.0/2I10/4I10)
  WRITE(6, 13) AI, B, KC, S, CA, NI, DN, NL, NPLK, IO, IF
  13 FORMAT(45X'CRACK PROPAGATION STUDY'///55X'INPUT'///30X'INITIAL CRACK
  1K DEPTH', 11X, AI = 'E20.8/30X'INITIAL CRACK WIDTH', 11X, B = 'E20.8/
  250X'FRACTURE TOUGHNESS', 12X, KC = 'E20.8/
  3 30X'EXPONENT OF DK, 16X, S = 'E20.8/30X'MATERIAL CONSCRACK
  4TANI', 13, CA = 'E20.8/30X'INITIAL CYCLE NUMBER', 10X, NI = 'E20.8/
  530X'CYCLE PRINT INTERVAL', 10X, DN = 'E20.8/30X'NUMBER LOAD CONDITIONCRACK
  6S, 8X, NL = 'E20.8/30X'NUMBER LOAD BLOCKS', 12X, LDS = 'E20.8/

```

```

730X*NUMBER OF G VALUES,12X*IC = 'I20/30X*NUMBER OF F VALUES',12X CRAC0420
8*IF = 'I20//) CRAC0430
94 INK = 121 CRAC0440
DATA(ABN(I),I=1,11)/0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0/ CRAC0450
DATA(CAT(I),I=1,11)/0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0/ CRAC0460
DATA(CMK(I),I=1,121)/1.1,1.1,1.1,1.1,1.1,1.1,1.1,1.1,1.1,1.1,1.1,1.1, CRAC0470
11.005,1.003,1.002,1.001,1.001,1.001,1.001,1.001,1.001,1.001,1.001,1.001, CRAC0480
21.021,1.01,1.007,1.005,1.003,1.001,1.001,1.001,1.001,1.001,1.001,1.001, CRAC0490
31.048,1.024,1.018,1.013,1.008,1.006,1.005,1.004,1.003,1.002,1.001,1.001, CRAC0500
41.091,1.042,1.034,1.025,1.018,1.014,1.011,1.008,1.006,1.004,1.003,1.003, CRAC0510
51.155,1.074,1.065,1.052,1.041,0.03,1.025,1.019,1.013,1.011,1.008,1.008, CRAC0520
61.25,1.119,1.107,1.091,1.077,1.065,1.051,1.043,1.035,1.029,1.024,1.024, CRAC0530
71.4,1.193,1.167,1.146,1.131,1.111,1.093,1.082,1.072,1.062,1.052,1.052, CRAC0540
81.667,1.349,1.279,1.245,1.217,1.191,1.168,1.148,1.131,1.117,1.102,1.102, CRAC0550
92.294,1.746,1.536,1.43,1.361,1.32,1.291,1.265,1.241,1.22,1.199,1.199, CRAC0560
13.1,2.35,1.915,1.7,1.58,1.53,1.49,1.455,1.42,1.39,1.36,1.36, CRAC0570
NL = NL + 1 CRAC0580
IF(MOTBL.LE.0) GO TO 27 CRAC0590
READ(5,25)(AB(I),00(I),I=1,10) CRAC0600
25 FORMAT(2F10.0) CRAC0610
IF(IQ.GT.1) GO TO 26 CRAC0620
WRITE(6,29) 00(1) CRAC0630
29 FORMAT(//20X*NOTE-- Q IS A CONSTANT = 'E15.8/') CRAC0640
GO TO 61 CRAC0650
27 WRITE(6,28) CRAC0660
28 FORMAT(//20X*NOTE-- THE A/B OR B/A VS. PHI(SQUARED) TABLE FOR THIS CRAC0670
1 CASE IS THE SAME AS THE PRECEDING CASE'//)
GO TO 61 CRAC0680
26 WRITE(6,35) (AB(I),00(I),I=1,10) CRAC0690
35 FORMAT (//43X'A/B OR B/A*9X'PHI(SQUARED)'/((34X2E20.8//)) CRAC0700
61 CONTINUE CRAC0710
IF(MFIBL.LE.0) GO TO 55 CRAC0720
READ(5,56)(AAB(I),F1(I),I=1,1F) CRAC0730
56 FORMAT(2F10.0) CRAC0740
IF(IQ.GT.1) GO TO 60 CRAC0750
WRITE(6,57) F1(1) CRAC0760
57 FORMAT(//20X*NOTE-- CORRECTION FACTOR F IS A CONSTANT = 'E15.8/') CRAC0770
GO TO 62 CRAC0780
55 WRITE(6,58) CRAC0790
58 FORMAT(//20X*NOTE-- THE A/B VS. F TABLE FOR THIS CASE IS THE SAME CRAC0800
1AS THE PRECEDING CASE'//)
GO TO 62 CRAC0810
60 WRITE(6,59)(AAB(I),F1(I),I=1,1F) CRAC0820
59 FORMAT(//47X'A/B',20X'F'/(34X2E20.8//)) CRAC0830
CRAC0840
CRAC0850

```

62 CONTINUE	CRAC0860
READ (5,4) (NF(I),DP(I),R(I),HH(I),PRC(I),ISC(I),I=2,NL)	CRAC0870
4 FORMAT (5F10.0,I10)	CRAC0880
DO 20 JB=1,NBLK	CRAC0890
IF(JB.EQ.1) GO TO 30	CRAC0900
WRITE(6,101) JB	CRAC0901
101 FORMAT(//55X'OUTPUT'/'48X'BLOCK LOAD NO.'I5//)	CRAC0902
A1 = A(1)	CRAC0910
B = A(2)	CRAC0920
50 CONTINUE	CRAC0930
NF(1) = NI	CRAC0940
IF(JB.GT.1) GO TO 102	CRAC0941
WRITE (6,6) JB	CRAC0950
6 FORMAT (1H1,55X'OUTPUT' /48X'BLOCK LOAD NO.' I5//)	CRAC0960
102 IQUIT = 0	CRAC0970
A(1) = A1	CRAC0980
A(2) = B	CRAC0990
DO 20 K=2,NL	CRAC1000
H = HH(K)	CRAC1010
PREC = PRC(K)	CRAC1020
IDONT = ISC(K)	CRAC1030
PT = -H	CRAC1040
N = 2	CRAC1050
IF(TYPE.LT.2.0) N=1	CRAC1051
NB(1) = 0	CRAC1060
NB(2) = 0	CRAC1070
MIN = 3	CRAC1080
IP = 1	CRAC1090
INIT = 0	CRAC1100
XN = NF(K-1)	CRAC1110
IF (NF(K) .GT. NF(K-1)) GO TO 8	CRAC1120
XN = NI	CRAC1130
A(1) = A1	CRAC1140
A(2) = B	CRAC1150
GO TO 9	CRAC1160
8 IF (IQUIT .EQ. 1) GO TO 500	CRAC1170
9 CONTINUE	CRAC1180
IF(JB.GT.1) GO TO 47	CRAC1190
WRITE (6,17)NF(K),DP(K),R(K),H,PREC,IDONT,START	CRAC1200
17 FORMAT (//1X'FINAL CYCLE NUMBER',7X,NF =E20.8/1X'CYCLIC STRESS RACRAC1210	
1NGE',6X'OP =E20.8/1X'CYCLIC STRESS PATIO',6X'R =E20.8/1X'STEP SCRAC1220	
21ZE',16X'H =E20.8/1X'PRECISION',16X'PRC=E20.8/1X'STEP CHANGE', CRAC1230	
314X'ISC=E120/1X'START CYCLE PRINT',6X'START=E18.9//)	CRAC1240
IF (TYPE.LT.2.0) GO TO 82	CRAC1250

```

WRITE(6,M0)
80 FOR=AI(9,M),14X'A',14X'B',12X'DA/DN',10X'DB/DN',10X'DK(A),
110X'DK(B),12X'Q'//)
GO TO 10
82 WRITE(6,M3)
83 FORMAT(9X'N',14X'A',12X'DA/DN',10X'DK(A),13X'E'//)
GO TO 10
47 IF(A.GT.2 .AND. JB.GT.1) GO TO 18
IF(TYPE.LT.2.0) GO TO 84
WRITE(6,M0)
GO TO 10
84 WRITE(6,M3)
GO TO 10
11 IP = 0
10 IF (XM .GE. NF(K)) GO TO 19
PT = PT + H
CALL CRANE
IF(TYPE.LT.2.0) GO TO 95
IF (A(1).GE.T) GO TO 21
IF(CELOM).GT.0.0 .AND. DENOM2.GT.0.0) GO TO 15
GO TO 21
95 IF(CELOM1.GT.0.0) GO TO 15
21 CONTINUE
IGUIT = 1
GO TO 19
15 IF(JB.GT.1) GO TO 71
IF(XM.LE.0.0) GO TO 70
GO TO 71
70 IF(TYPE.LT.2.0) GO TO 73
WRITE(6,16)X:A(1),A(2),FF(1),FF(2),DK1,DK2,0
GO TO 71
73 WRITE(6,66)X:A(1),FF(1),DK1,F
71 IF (PT .LT. DN) GO TO 11
IF(IP.EQ.1) GO TO 18
IF(XM .LT. START) GO TO 18
IF(TYPE.LT.2.0) GO TO 85
WRITE (6,16)X:A(1),A(2),FF(1),FF(2),DK1,DK2,0
18 FORMAT (1X,6E15.8)
GO TO 18
85 WRITE(6,86)X:A(1),FF(1),DK1,F
86 FORMAT(1X,5E15.8)
19 IF (XM .GE. NF(K)) GO TO 20
PT = 0.0
GO TO 11

```

CRAC1260
CRAC1270
CRAC1280
CRAC1290
CRAC1300
CRAC1310
CRAC1320
CRAC1321
CRAC1330
CRAC1340
CRAC1350
CRAC1360
CRAC1370
CRAC1380
CRAC1390
CRAC1400
CRAC1410
CRAC1420
CRAC1430
CRAC1440
CRAC1450
CRAC1460
CRAC1470
CRAC1480
CRAC1490
CRAC1491
CRAC1500
CRAC1510
CRAC1520
CRAC1530
CRAC1540
CRAC1550
CRAC1560
CRAC1570
CRAC1580
CRAC1590
CRAC1600
CRAC1610
CRAC1620
CRAC1630
CRAC1640
CRAC1650
CRAC1660
CRAC1670

```

19 IF (TYPE.LT.2.0) GO TO 87
   WRITE (6,16) XN,A(1),A(2),FF(1),FF(2),DK1,DK2,Q
   GO TO 20
87 WRITE(6,16) XN,A(1),FF(1),DK1,F
20 CONTINUE
   GO TO 500
END
CRAC1680
CRAC1690
CRAC1700
CRAC1710
CRAC1720
CRAC1730
CRAC1740

```

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10 FOR DERIV
  SUBROUTINE DERIV
    DIMENSION MMK(200), AB(100), QQ(100), AT(100), R(100), DP(100)
    1,ABN(100),FI(100),AAB(100)
    REAL W,KC,MMK
    COMMON /CRANE/ XN,A(40),FF(40),H,N,I,P(40),MIN,PRFC,ICNT,INIT
    COMMON /CRAC/ K,B,I,IMK,IQ,AB,AT,MMK,QQ,DP,R,S,KC,CA,G,DENOM1
    1,Y,A1,DENOM2,DK1,DK2,Q,ADT,ADB,M,ABM,F1,IF,AAB,TYPE,F,CB
    AA = A(1)
    BB = A(2)
    M = 1.0
    IF (TYPE.LT.2.0) GO TO 10
    ADT = AA/T
    ADB = AA/BB
    IF (BB.LT.AA) ADB=BB/AA
    IF (G.LE.1.0) GO TO 5
    IF (AA.LE.BB) CALL DISCOT(ADB,ADT,ABM,MMK,AT,+022,IMK,11,M)
    Q = QA(1)
    IF (IQ.GT.1) CALL DISCOT(ADB,ADB,AB,QQ,QQ,-020,IQ,0,Q)
    IF (IC.GT.1) Q=Q-0.212*(DP(K)/(Y*(1.0-R(K))))**2
    AOB = AA/BB
    F = 1.0
    IF (AOB.LE.2.0) CALL DISCOT(AOB,AOB,AAB,F1,F1,-020,IF,Q,F)
    GO TO 20
5 IF (AA.LE.BB) CALL DISCOT(ADB,ADT,ABM,MMK,AT,+022,IMK,11,M)
    Q = QA(1)
    IF (IQ.GT.1) CALL DISCOT(ADB,ADB,AB,QQ,QQ,-020,IQ,0,Q)
    IF (IC.GT.1) Q=Q-0.212*(DP(K)/(Y*(1.0-R(K))))**2
    F = F1(1)
20 DK1 = F*DP(K)*BB/AA*SQRT(3.1415926*AA/Q)
    IF (AA.LE.BB) DK1 = F*M*DP(K)*SQRT(3.1415926*AA/Q)
    DK2 = G*DP(K)*SQRT(3.1415926*BB/Q)
    IF (AA.LE.BB) DK2 = G*DP(K)*SQRT(3.1415926*BB/Q)*(AA/BB)
    DERI0000
    DERI0010
    DERI0020
    DERI0030
    DERI0040
    DERI0050
    DERI0060
    DERI0070
    DERI0080
    DERI0090
    DERI0100
    DERI0110
    DERI0120
    DERI0130
    DERI0140
    DERI0150
    DERI0160
    DERI0170
    DERI0180
    DERI0190
    DERI0200
    DERI0210
    DERI0220
    DERI0230
    DERI0240
    DERI0250
    DERI0260
    DERI0270
    DERI0280
    DERI0290
    DERI0300
    DERI0310
    DERI0320

```



```

DENOM1=(1.0-R(K))*KC-DK1
FF(1)=CA*DK1**S/DENOM1
DENOM2=(1.0-R(K))*KC-DK2
FF(2)=CB*DK2**S/DENOM2
GO TO 25
10 AOB = AA/BB
F = 1.0
IF(AOB.LE.2.0) CALL DISCOT(AOB,AOB,AAB,F1,-020,IF,0,F)
Q = QR(1)
DK1 = F*DP(K)*M*SQRT(3.1415926*AA/0)
DENOM1 = (1.0-R(K))*KC-DK1
FF(1)=CA*DK1**S/DENOM1
25 RETURN
END

```

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C   FOR CRANE
C   SUBROUTINE CRANE
C   THE ARGUMENTS OF THIS SUBROUTINE ARE ALL CONTAINED IN THE FOLLOW-
C   ING COMMON STATEMENT, LABELED CRANE. DRIVER PROGRAM MUST CONFORM
C   IN ORDER AND DIMENSIONS WITHIN COMMON/CRANE/, THOUGH NOT NECES-
C   SARILY IN THE NAMES OF THE VARIABLES.
C   COMMON/CRANE/X,Y(40),F(40),H,N,NR(40),MIN,PREC,IDONT,INIT
C   MEANING OF THE ARGUMENTS ARE AS FOLLOWS
C   X   THE INDEPENDENT VARIABLE.
C   Y   ARRAY OF DEPENDENT VARIABLES, 40 MAXIMUM.
C   F   ARRAY OF DERIVATIVES, FUNCTIONS OF X AND THE Y VECTOR.
C   H   STEP SIZE IN X. MUST BE NON-ZERO. USUALLY MAY BE ALTERED
C   INTERNALLY BY CRANE, AFTER FIRST BEING SET + OR - EXTERNALLY.
C   N   THE NO. OF Y S AND F S IN THE SYSTEM. USUALLY THE NO. ALSO
C   OF COUPLED ORDINARY DIFF. EQUNS.
C   NR   AN ARRAY OF FLAGS. IF +, HOLD ABSOLUTE TRUNCATION ERROR EST.
C   BELOW A SPECIFIED LIMIT = 10.**(-PREC)
C   IF 0, DITTO FOR RELATIVE TRUNCATION ERROR
C   ESTIMATE.
C   IF -, DISREGARD THE TRUNC. ERROR.
C   FOR THE Y S WHOSE FLAGS ARE NEGATIVE.
C   MIN   THE MINIMUM NO. OF STEPS BETWEEN OCCASIONS FOR DOUBLING H.
C   THE VALUE OF MIN MAY NOT EXCEED 10, NOR BE LESS THAN 3.
C   INIT SET TO ZERO BEFORE FIRST CALL, CAUSING THE SR TO INITIALIZE

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C      ITSELF. THEREAFTER, INIT IS BUMPED INTERNALLY, EACH STEP. CRAN0260
C      IDONT A FLAG WHICH IF NON-ZERO, FORBIDS ANY CHANGE IN H TO BE CRAN0270
C      MADE INTERNALLY BY THE SP. CRAN0280
C      PREC A POSITIVE CONST. APPROXIMATELY EQUAL TO THE NO. OF SIGNIFI- CRAN0290
C      CANT DECIMAL DIGITS OF LOCAL PRECISION. (BASED ON TRUNCATION CRAN0300
C      ERROR ESTIMATES MADE INTERNALLY. EFFECTIVE ONLY IF IDONT=0.) CRAN0310
C
C      CRAN0320
C      THE PARTICULAR PREDICTOR-CORRECTOR ALGORITHM IS THAT PUBLISHED CRAN0330
C      BY CRANE AND KLOPFENSTEIN IN J.A.C.M. VOL 12, PAGES 227-241, APRIL CRAN0340
C      1965. IT IS OF FOURTH ORDER, I.E., TRUNCATION ERRORS ARE OF ORDER CRAN0350
C      H**5. THE ALGORITHM WAS DEVELOPED TO MAXIMIZE THE RANGE OF STEP CRAN0360
C      SIZE CONSISTENT WITH ABSOLUTE STABILITY, AND IN A LOOSER SENSE, TO CRAN0370
C      HAVE A GOOD RANGE OF RELATIVE STABILITY. THESE RANGES, EXPRESSED CRAN0380
C      AS H, NORMALIZED BY MULTIPLYING BY PARTIAL DERIV OF Y(=G(Y,Y)) WITH CRAN0390
C      RESPECT TO Y, ARE CRAN0400
C      FOR ABSOLUTE, 0.6E.HBAR.GF.(-2.4909) CRAN0410
C      FOR RELATIVE, INFINITY.GE.HBAR.GE.(-.496) CRAN0420
C      THE STARTING PROCEDURE IS THE RUNGE-KUTTA VARIANT PUBLISHED BY CRAN0430
C      S. GILL, CAMB. PHIL. SOC. PROC., 47, P96,(1951) CRAN0440
C      CRAN0450
C      TO USE THIS ROUTINE, A SUBROUTINE CALLED DERIV MUST BE PROVI- CRAN0460
C      DED WHICH USES LABELED COMMON/CRANEC/AND CALCULATES THE F VECTOR. CRAN0470
C      THE FIRST CALL OF CRANE WILL ONLY OBTAIN F'S, AND SET INIT=1. EACH CRAN0480
C      SUBSEQUENT CALL WILL ADVANCE ONE STEP AND UPDATE X,Y AND F, EXCEPT CRAN0490
C      THAT AFTER THREE STEPS, CRANE MAY HALVE H AND RETURN TO INITIAL CRAN0500
C      CONDITIONS WITH INIT=4. (THIS BEHAVIOR WILL RECUR UNTIL PRECISION CRAN0510
C      IS SATISFIED ACCORDING TO AN ERROR ESTIMATE AFTER THREE STEPS.) CRAN0520
C      CRAN0530
C      PRIOR TO FIRST CALL, SET INIT X AND Y AND DEFINE THE VALUES CRAN0540
C      OF THE NB VECTOR. ALSO DEFINE VALUES OF N,MIN,INIT(=0),IDONT, AND CRAN0550
C      PREC. PREC SHOULD ADVISEDLY BE FROM 2.5 TO 7. (NOT NECESSARILY CRAN0560
C      AN INTEGER), THOUGH THERE WILL BE NO EFFECT IF IDONT.NE.0 CRAN0570
C      CRAN0580
C      PRINTOUT AND TERMINATION ARE NOT ACCOMPLISHED BY CRANE. (NOT) CRAN0590
C      CRAN0600
C      DO NOT CHANGE X,Y,F,N OR H EXTERNALLY UNLESS YOU SET INIT=0, CRAN0610
C      EXTERNALLY. HOWEVER, NB AND PREC MAY BE CHANGED AT ANY TIME. SET- CRAN0620
C      TING INIT.LT.0 WILL RESULT IN INHIBITING INCREASE IN H FOR MIN CRAN0630
C      STEPS, BUT WILL PERMIT DECREASES IN H. INIT WILL BE POSITIVE ON CRAN0640
C      EXITTING CRANE AGAIN. CRAN0650
C      DIMENSION G(40),Y1(40),Y2(40),F0(40),F1(40),F2(40),F3(40),F4(40), CRAN0660
C      1F5(40),Y2(40),E(10),Y3(40),Y4(40),Y5(40) CRAN0670
C      EQUIVALENCE(Y5(1),Q(1)),(R,T) CRAN0680
C      Y5 AND Q ARE NOT BOTH NEEDED AT THE SAME INSTANT. CRAN0690

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C	DATA KTRU/0/	CRAN0700
	FIRST EXECUTABLE NEXT	CRAN0710
	IF (INIT) 1,2,3	CRAN0720
	1 INIT=0	CRAN0730
	E (MIN)=10.	CRAN0740
	IF (KTRU) 16,2,16	CRAN0750
	2 IF (N) 199,199,555	CRAN0760
	555 IF (N-40) 5,5,199	CRAN0770
	199 WRITE (6,200)	CRAN0780
	200 FORMAT (54H0N WAS NOT A POSITIVE INTEGER,LT.40, ON CALLING CRANE.)	CRAN0790
	CALL EXIT	CRAN0800
	5 IF (MIN-10) 6,7,8	CRAN0810
	8 MIN=10	CRAN0820
	6 IF (MIN-3) 9,7,7	CRAN0830
	9 MIN=3	CRAN0840
	7 KTRU=0	CRAN0850
	70 IF (PREC-7.) 11,13,12	CRAN0860
	12 PREC=7.	CRAN0870
	11 IF (PREC-2.0) 14,13,13	CRAN0880
	14 PREC=4.	CRAN0890
C	STATEMENT 14 ASSUMES THAT THE OVERSIGHT OF SPECIFYING NO PRECI-	CRAN0900
C	SION, OR PRECISION BELOW 2.0 DECIMALS, SHOULD CALL FOR 4 DECIMALS, CRAN0910	CRAN0910
C	VIA INTERNAL CORRECTION. ALSO, PREC ABOVE 7. IS REDUCED TO 7.	CRAN0920
	13 PREC=PREC	CRAN0930
	T=10.**(-PREC)	CRAN0940
	BOUND=T*16.21966	CRAN0950
	BOUND=T*.05	CRAN0960
	BOUND=BOUND*FLOAT(MIN)	CRAN0970
	IF (INIT) 1155,1155,16	CRAN0980
	1155 CALL DERIV	CRAN0990
	17 K=0	CRAN1000
	ASSIGN 55 TO IS	CRAN1010
	H1=5*H	CRAN1020
	KBC=3	CRAN1030
	IF (KTRU) 18,18,20	CRAN1040
	15 CALL DERIV	CRAN1050
	18 INIT=1+INIT	CRAN1060
	19 RETURN	CRAN1070
C	THE ABOVE IS THE ONLY RETURN FROM INTODE. NEXT IS REACHED WHEN	CRAN1080
C	INIT EXCEEDS 0.	CRAN1090
	3 IF (PREC-PREC) 70,16,70	CRAN1100
	16 IF (K) 20,20,21	CRAN1110
	20 K=4	CRAN1120
	213 DO 22 I=1,N	CRAN1130

F3(I)=F2(I)	CRAN1140
F2(I)=F1(I)	CRAN1150
F1(I)=F0(I)	CRAN1160
F0(I)=F(I)	CRAN1170
Y3(I)=Y2(I)	CRAN1180
Y2(I)=Y1(I)	CRAN1190
Y1(I)=Y0(I)	CRAN1200
Y0(I)=Y(I)	CRAN1210
22 IF(RBC)23,23,24	CRAN1220
24 KBC=KBC-1	CRAN1230
XN=X+H	CRAN1240
X=X+H1	CRAN1250
RX=.292893219	CRAN1260
ASSIGN 33 TO KS	CRAN1270
37 DO 29 I=1,N	CRAN1280
AK=H*F(I)	CRAN1290
GO TO KS,(33,34,30)	CRAN1300
33 Y(I)=Y(I)+0.5*AK	CRAN1310
Q(I)=AK	CRAN1320
GO TO 29	CRAN1330
30 Y(I)=(-Q(I)-Q(I)+AK)*.166666667+Y(I)	CRAN1340
GO TO 29	CRAN1350
34 R=RX*(AK-G(I))	CRAN1360
Q(I)=(R+R)-RX*AK)+Q(I)	CRAN1370
Y(I)=Y(I)+R	CRAN1380
29 CONTINUE	CRAN1390
GO TO (1551, 224, 219, 251),K	CRAN1400
219 RX=1.70710678	CRAN1410
GO TO 1551	CRAN1420
251 ASSIGN 34 TO KS	CRAN1430
GO TO 1551	CRAN1440
224 ASSIGN 30 TO KS	CRAN1450
X=XN	CRAN1460
1551 CALL DERIV	CRAN1470
36 K=K-1	CRAN1480
IF(K)18,18,37	CRAN1490
C THE LOOP BEGINNING AT 37 IS EXCTD 4 TIMES BEFORE ONE STEP IN	CRAN1500
C DELTAX IS DONE. THE CODING IS A BIT COMPLEX IN ORDER TO MINIMIZE	CRAN1510
C MULTIPLICATIONS AND INDEXING. IT IS EQUIVALENT TO SLOWER CODE,	CRAN1520
C AS FOLLOWS... DO 36 J=1,4 WHEREIN J=5-K	CRAN1530
C DO 29 I=1,N+1	CRAN1540
C AK(I,J)=DELTAX*F(I)	CRAN1550
C R(I,J)=A(J)*AK(I,J)-B(J)*Q(I)	CRAN1560
C Y(I)=Y(I)+R(I,J)	CRAN1570

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C      Q(I) =Q(I)+3.* R(I,J)-C(J)*AK(I,J)      CRAN1580
C      29 X= Y(N+1)      CRAN1590
C      CALL DERIV (X,Y,F)      CRAN1600
C      36 CONTINUE      CRAN1610
C      RETURN      CRAN1620
C      ACC. TO S. GILL, CAMB. PHIL. SOC., PROC., 47,P96 (1951), THE      CRAN1630
C      VALUES OF A, U, AND C ARE REST....      CRAN1640
C      A(J)=0.5,1.-SORTF(.5),1.+SORTF(.5),1./6.      J=1,4      CRAN1650
C      B(J)=1.,1.-SORTF(.5),1.+SORTF(.5),1./3.      J=1,4      CRAN1660
C      C(J)=0.5,1.-SORTF(.5),1.+SORTF(.5),0.5      J=1,4      CRAN1670
C      CRAN1680
C      FOLLOWING IS REACHED FROM SIMT 16 WHEN KBC=0, AFTER PRFD-CORR      CRAN1690
C      INITIALIZATION IS COMPLETE.      CRAN1700
C      21 IF(IDONT)211,212,211      CRAN1710
C      211 KBC=0      CRAN1720
C      GO TO 213      CRAN1730
C      212 IF(E(MIN)-BOUNDA)100,100,28      CRAN1740
C      100 DO 4 I=2,MIN      CRAN1750
C      4 E(I)=E(I)+E(I)      CRAN1760
C      IF(E(I)-BOUND)38,28,28      CRAN1770
C      28 IS USUAL. 38 IS PREPARE TO DOUBLE DELTAX.      CRAN1780
C      28 DO 39 I=1,N      CRAN1790
C      F5(I)=F4(I)      CRAN1800
C      F4(I)=F3(I)      CRAN1810
C      F3(I)=F2(I)      CRAN1820
C      F2(I)=F1(I)      CRAN1830
C      F1(I)=F0(I)      CRAN1840
C      F0(I)=F(I)      CRAN1850
C      Y5(I)=Y4(I)      CRAN1860
C      Y4(I)=Y3(I)      CRAN1870
C      Y3(I)=Y2(I)      CRAN1880
C      Y2(I)=Y1(I)      CRAN1890
C      Y1(I)=Y0(I)      CRAN1900
C      Y0(I)=Y(I)      CRAN1910
C      39 Y(I)=1.547652*Y(I)+2.017204*Y2(I)-1.857503*Y1(I)-.697353*Y3(I)+S*      CRAN1920
C      1(.965508124*F(I)+.895121303*F2(I)-.35158071*F3(I)-F1(I))      CRAN1930
C      40 X=X+H      CRAN1940
C      CALL DERIV      CRAN1950
C      42 DO 10 I=2,MIN      CRAN1960
C      10 E(I-1)=E(I)      CRAN1970
C      E(MIN)=0.      CRAN1980
C      DO 43 I=i,N      CRAN1990
C      T=Y0(I)+AK*(9.*F(I)+19.*F0(I)+F2(I)-5.*F1(I))      CRAN2000
C      PNC=Y(I)-T      CRAN2010

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C	AT THIS POINT, PMC IS PREDICTOR-CORRECTOR	CRAN2020
	IF(IGINT)43,72,43	CRAN2030
	72 IF(GB(I))43,44,47	CRAN2040
	44 IF(Y(I))46,43,46	CRAN2050
	46 PMCEPMC/Y(I)	CRAN2060
	47 IF(ABS(PMC)-E(MIN))43,43,48	CRAN2070
	48 E(MIN)=ABS(PMC)	CRAN2080
	43 Y(I)=T	CRAN2090
C	HERE, E(MIN) IS THE MAX. TRUNC. ERROR EST., ARG. OR DEL., OVER	ICRAN2100
C	WHILE THIS REST OF E ARE ITS PREDECESSORS.	CRAN2110
	IF(E(MIN)-BOUND)50,50,51	CRAN2120
	50 GO TO 15,(15,55)	CRAN2130
C	15 IS NORMAL AND CALLS DERIV, JUMPING TO 18 TO STEP INIT+1, RETURN	CRAN2140
C	55 IS REACHED FIRST STEP AFTER A SEQUENCE OF RKG STEPS.(OPD 0510)	CRAN2150
C	WHEN LARGEST ERROR EXCEEDS BOUND, STMT 51 IS A BRANCH GOVERNED	CRAN2160
C	BY WHETHER THIS IS ON FIRST P-C STEP OR NOT. 52 IS NORMAL.	CRAN2170
	51 IF(KBC)59,59,52	CRAN2180
C	52 IS REACHED WHEN LARGEST ERROR EXCEEDS BOUND, FNCTN OF PRECISN.	CRAN2190
	52 XEX-H	CRAN2200
	DO 54 I=1,N	CRAN2210
	Y(I)=Y0(I)	CRAN2220
	54 F(I)=F0(I)	CRAN2230
	53 H=.625*H	CRAN2240
	GO TO 17	CRAN2250
C	RETURN TO 17 SETS UP RKG INITIALIZATION	CRAN2260
	55 KBC=3	CRAN2270
	ASSIGN 15 TO 15	CRAN2280
	KTRU=1	CRAN2290
	GO TO 15	CRAN2300
C	ABOVE SEQUENCE COMPLETES INITIALIZATION OF PRED-CORR ROUTINE.	CRAN2310
C	NEXT, FAILURE OF FIRST TEST AFTER RKG SEQUENCE CAUSES BACKUP 3	CRAN2320
C	STEPS.	CRAN2330
	59 XEX-4.*H	CRAN2340
	DO 60 I=1,N	CRAN2350
	Y(I)=Y3(I)	CRAN2360
	60 F(I)=F3(I)	CRAN2370
	GO TO 53	CRAN2380
C	REFER TO CARD 1130 FOR RETURN TO RKG INITIALIZATION.	CRAN2390
C	THE FOLLOWING SETS UP DOUBLING DELTX	CRAN2400
	58 H=2.*H	CRAN2410
	DO 64 I=1,N	CRAN2420
	F2(I)=F3(I)	CRAN2430
	F3(I)=F5(I)	CRAN2440
	F0(I)=F(I)	CRAN2450

```

Y2(I)=Y3(I)
Y3(I)=Y5(I)
64 Y0(I)=Y(I)
C 23 IS REACHED AFTER 3 RKG STEPS, I E WHEN KBC=0.(REF. STTMT 22)
23 AK=.0416666667*H
E(MIN)=10.
S=H*2.03169
DO 62 I=1,N
62 Y(I)=1.547652*Y(I)+2.017204*Y2(I)-1.867503*Y1(I)-.69753*Y3(I)+S*
1(.965508124*F(I)+.895121303*F2(I)-.351589071*F3(I)-F1(I))
60 TO 40
C ABOVE CALLS DERIVATIVE WITH FIRST PREDICTED Y VECTOR.
END
CRAN2460
CRAN2470
CRAN2480
CRAN2490
CRAN2500
CRAN2510
CRAN2520
CRAN2530
CRAN2540
CRAN2550
CRAN2560
CRAN2570
CRAN2580

```

```

@ FOR DISCOT
SUBROUTINE DISCOT (XA,ZA,TABX,TABY,TABZ,NC,NY,NZ,ANS)
DIMENSION TABX(500),TABY(500),TABZ(500),NPX(8),NPY(8),YY(8)
C DIMENSION TABX(500),TABY(500),TABZ(500),NPX(8),NPY(8),YY(8)
CALL UNS (NC,IA,IDX,IDZ,IMS)
IF (NZ-1) 5,5,10
5 CALL DISSER (XA,TABX(1),1,NY,IDX,NN)
NNN=IDX+1
CALL LAGRAN (XA,TABX(NNN),TABY(NNN),NNN,ANS)
GOTO 70
10 ZARG=7A
IP1X=IDX+1
IP1Z=IDZ+1
IF (IA) 15,25,15
15 IF (ZARG-TABZ(NZ)) 25,25,20
20 ZARG=TABZ(NZ)
25 CALL DISSER (ZARG,TABZ(1),1,NZ,IDZ,NPZ)
NX=NY/NZ
NPZL=NPZ+IDZ
I=1
IF (IMS) 30,30,40
30 CALL DISSER (XA,TABX(1),1,NY,IDX,NPX(1))
DO 35 JJ=NPZ,NPZL
NPY(I)=(JJ-1)*NX+NPX(1)
NPX(I)=IPX(1)
35 I=I+1
GOTO 50
DISC0000
DISC0010
DISC0020
DISC0030
DISC0040
DISC0050
DISC0060
DISC0070
DISC0080
DISC0090
DISC0100
DISC0110
DISC0120
DISC0130
DISC0140
DISC0150
DISC0160
DISC0170
DISC0180
DISC0190
DISC0200
DISC0210
DISC0220
DISC0230
DISC0240
DISC0250
DISC0260

```



```

      RETURN
      40 IPX=NDIS+1
      RETURN
      END
DISS0280
DISS0290
DISS0300
DISS0310

```

```

10 FOR UNS
  SUBROUTINE UNS (IC,IA,IDX,IDZ,IMS)
    IF (IC) 5,5,10
    5 IMS=1
      NC=-IC
      GOTO 15
    10 IMS=0
      NC=IC
    15 IF (NC-100) 20,25,25
    20 IA=U
      GOTO 30
    25 IA=1
      NC=NC-100
    30 IDX=NC/10
      IDZ=NC-IDX*10
      RETURN
      END
UNS 0000
UNS 0010
UNS 0020
UNS 0030
UNS 0040
UNS 0050
UNS 0060
UNS 0070
UNS 0080
UNS 0090
UNS 0100
UNS 0110
UNS 0120
UNS 0130
UNS 0140
UNS 0150
UNS 0160

```

```

10 FOR LAGRAN
  SUBROUTINE LAGRAN (XA,X,Y,N,ANS)
    DIMENSION X(200),Y(200)
    C
    DIMENSION X(200),Y(200)
    SUM=0.0
    DO 3 I=1,N
      PROD=Y(I)
      DO 2 J=1,N
        A=X(I)-X(J)
        IF (A) 1,2,1
      1 B=(XA-X(J))/A
      PROD=PROD*B
    2 CONTINUE
    3 SUM=SUM+PROD
    ANS=SUM
    RETURN
    END
LAGR0000
LAGR0010
LAGR0020
LAGR0030
LAGR0040
LAGR0050
LAGR0060
LAGR0070
LAGR0080
LAGR0090
LAGR0100
LAGR0110
LAGR0120
LAGR0130
LAGR0140
LAGR0150
LAGR0160

```

5/10/10 Roll off roll-off 10/10/10

APPENDIX C
INPUT AND PRINTOUT FOR
SAMPLE PROBLEM

INPUT FOR SAMPLE PROBLEM

CARD 1

Title of problem

TWO-DIMENSIONAL SURFACE FLAW: SPECIMEN NO. 9L (2219-T87 ALUMINUM)																																																																															
C- FOR COMMENT										CONTINUATION																																																												IDENTIFICATION									
STATEMENT										FORTF AN STATEMENT																																																																					
NUMBER																																																																															
1 000										0000 000000 0000000000 000 0000000000000000 00000 00000 000 00000000																																																												00000000									
12 3 4 5 6										7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72																																																												73 74 75 76 77 78 79 80									

CARD 2

Type of flaw: 1.0 = one-dimensional; 2.0 = two-dimensional

[illegible]

CARD 3

Initial crack depth	AI = 0.0660
Plate thickness	T = 0.1259
Initial crack half-width	B = 0.1240

C FOR COMMENT		STATEMENT NUMBER		CONTINUATION		FORTRAN STATEMENT		IDENTIFICATION	
0	0	0	0	0	0	0	0	0	0
1	2	3	4	5	6	7	8	9	0
1	2	3	4	5	6	7	8	9	0

CARD 4

Fracture toughness	KC = 40000.0
Material yield stress	Y = 55500.0
Exponent of DK	S = 2.5
Correction factor	G = 1.12
Material constant for A	CA = 0.14×10^{-10}
Material constant for B	CB = 0.28×10^{-10}

[illegible]

```
Initial cycle number      NI = 0.0
Cycle print interval      DN = 10.0
Start cycle print        START = 40000.0
Number load conditions    NL = 1
Number load blocks        NBLK = 1
```

CARD 6

Number of Q values	IQ = 21
Flag for reading Q table	MQTBL = 1
Number of F values	IF = 12
Flag for reading F table	MFTBL = 1

CARD 7 (21 cards) (only the first card is shown.)

21 values of A/B compared with Φ^2 from table IV.

CARD 8 (12 cards) (only the first card is shown.)

12 values of A/B compared with F from table I.

50

1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

```

NF = 40000.0
DP = 19570.0
R = 0.01
HH = 10.0
PRC = 7.0
ISC = 0

```

|| | | | |

067 0 13

21

[illegible]

CARD 8

These cards were not entered for this problem because they were read in for the first problem.

CARD 9

[illegible]

PRINTOUT FOR SAMPLE PROBLEM

TWO DIMENSIONAL SURFACE FLAW. SPECIMEN NO. 9L (2219-787 ALUMINUM)

CRACK PROPAGATION STUDY

INPUT

INITIAL CRACK DEPTH AT = .66000000+01
 PLATE THICKNESS T = .12590000+00
 INITIAL CRACK WIDTH B = .12400000+00
 FRACTURE TOUGHNESS KC = .40000000+05
 MATERIAL YIELD STRESS Y = .55500000+05
 EXPONENT OF DK S = .25000000+01
 CORRECTION FACTOR G = .11200000+01
 MATERIAL CONSTANT FOR A CA = .14000000+10
 MATERIAL CONSTANT FOR B CB = .28000000+10
 INITIAL CYCLE NUMBER NI = .00000000
 CYCLE PRINT INTERVAL DN = .10000000+02
 NUMBER LOAD CONDITIONS NL = 1
 NUMBER LOAD BLOCKS LDB = 1
 NUMBER OF Q VALUES IQ = 21
 NUMBER OF F VALUES IF = 12

A/B OR B/A	PHI(SQUARED)
.00000000	.10000000+01
.22361000+00	.11246050+01
.31622000+00	.12205270+01
.38729000+00	.13073540+01
.44721000+00	.13988380+01
.50000000+00	.14666560+01
.54772000+00	.15417460+01
.59161000+00	.16147720+01
.63245000+00	.16859150+01
.67082000+00	.17555680+01
.70710000+00	.18242390+01

OUTPUT
BLOCK LOAD NO. 1

FINAL CYCLE NUMBER NF = .40000000+05
CYCLIC STRESS RANGE DP = .19570000+05
CYCLIC STRESS RATIO R = .10000000-01
STEP SIZE H = .10000000+01
PRECISION PRC = .70000000+01
STEP CHANGE ISC = 0
START CYCLE PRINT START = .40000000+05

N A B DA/DN DB/DN DK(A) DK(B) Q

.00000000 .66000000-01 .12400000+00 .25703528-05 .22856161-05 .80390698+04 .59630026+04 .14910896+01
.13253000+05 .12573194+00 .17348037+00 .12091028-04 .63389171-05 .13781761+05 .86713588+04 .18349097+01

TWO DIMENSIONAL SURFACE FLAW, SPECIMEN NO. 10L (2219-T87 ALUMINUM)

CRACK PROPAGATION STUDY

INPUT

INITIAL CRACK DEPTH AT = .67000000+01
PLATE THICKNESS T = .12450000+00
INITIAL CRACK WIDTH B = .12650000+00
FRACTURE TOUGHNESS KC = .40000000+05
MATERIAL YIELD STRESS Y = .55500000+05
EXPONENT OF DK S = .25000000+01
CORRECTION FACTOR G = .11200000+01
MATERIAL CONSTANT FOR A CA = .14000000-10
MATERIAL CONSTANT FOR B CB = .28000000-10
INITIAL CYCLE NUMBER NI = .00000000
CYCLE PRINT INTERVAL ON = .10000000+02
NUMBER LOAD CONDITIONS NL = 1
NUMBER LOAD BLOCKS LOS = 1
NUMBER OF Q VALUES IQ = 21
NUMBER OF F VALUES IF = 12

NOTE-- THE A/B OR P/A VS. PHI(SQUARED) TABLE FOR THIS CASE IS THE SAME AS THE PRECEDING CASE

NOTE-- THE A/B VS. F TABLE FOR THIS CASE IS THE SAME AS THE PRECEDING CASE

OUTPUT
BLOCK LOAD NO. 1

FINAL CYCLE NUMBER NF = .40000000+05
CYCLIC STRESS RANGE DP = .27690000+05
CYCLIC STRESS RATIO R = .10000000-01
STEP SIZE H = .10000000+01
PRECISION PRC = .70000000+01
STEP CHANGE ISC = 0
START CYCLE PRINT START = .40000000+05

N A B DA/DN DB/DN DK1A1 DK1B1 6

.00000000 .67000000-01 .12650000+00 .72971719-05 .61285903-05 .11627885+05 .85666760+04 .14610538+01
.44350000+04 .12460623+00 .17080691+00 .35934469+04 .17200877-04 .19492410+05 .12278008+05 .18160856+01

TWO DIMENSIONAL SURFACE FLAW: SPECIMEN NO. 12M (2219-187 ALUMINUM)

CRACK PROPAGATION STUDY

INPUT

INITIAL CRACK DEPTH AT = .05000000-01
PLATE THICKNESS T = .12550000+00
INITIAL CRACK WIDTH B = .13150000+00
FRACTURE TOUGHNESS KC = .40000000+05
MATERIAL YIELD STRESS Y = .55500000+05
EXPONENT OF DK S = .28000000+01
CORRECTION FACTOR G = .11200000+01
MATERIAL CONSTANT FOR A CA = .14000000-10
MATERIAL CONSTANT FOR B CB = .28000000-10
INITIAL CYCLE NUMBER NI = .00000000
CYCLE PRINT INTERVAL DN = .10000000+02
NUMBER LOAD CONDITIONS NL = 1
NUMBER LOAD BLOCKS LBS = 1
NUMBER OF Q VALUES IQ = 21
NUMBER OF F VALUES IF = 12

NOTE-- THE A/R OR B/A VS. PHI(SQUARED) TABLE FOR THIS CASE IS THE SAME AS THE PRECEDING CASE

NOTE-- THE A/R VS. F TABLE FOR THIS CASE IS THE SAME AS THE PRECEDING CASE

OUTPUT

BLOCK LOAD NO. 1

FINAL CYCLE NUMBER NF = .40000000+05
 CYCLIC STRESS RANGE DP = .31540000+05
 CYCLIC STRESS RATIO R = .10000000+01
 STEP SIZE H = .10000000+01
 PRECISION PRC = .70000000+01
 STEP CHANGE ISC = 0
 START CYCLE PRINT START = .40000000+05

N	A	B	DA/DN	DB/DN	DK(A)	DK(B)	Q
.00000000	.65000000+01	.13150000+00	.11021573+04	.82219570+05	.13365759+05	.95161298+04	.13908795+01
.30000000+04	.12552913+00	.17423380+00	.62553056+04	.25928876+04	.22536126+05	.19101255+05	.17829959+01

TWO DIMENSIONAL SURFACE FLAW. SPECIMEN NO. 14M (2219-167 ALUMINUM)

CRACK PROPAGATION STUDY

INPUT

INITIAL CRACK DEPTH AT = .63000000+01
 PLATE THICKNESS T = .11930000+00
 INITIAL CRACK WIDTH B = .12850000+00
 FRACTURE TOUGHNESS KC = .40000000+05
 MATERIAL YIELD STRESS Y = .55500000+05
 EXPONENT OF DK S = .25000000+01
 CORRECTION FACTOR G = .11200000+01
 MATERIAL CONSTANT FOR A CA = .14000000+10
 MATERIAL CONSTANT FOR B CB = .28000000+10
 INITIAL CYCLE NUMBER NI = .00000000
 CYCLE PRINT INTERVAL DN = .10000000+02
 NUMBER LOAD CONDITIONS NL = 1
 NUMBER LOAD BLOCKS LBS = 1
 NUMBER OF Q VALUES IQ = 21
 NUMBER OF F VALUES IF = 12

NOTE-- THE A/R OF R/A VS. PHI(SQUARED) TABLE FOR THIS CASE IS THE SAME AS THE PRECEDING CASE

NOTE-- THE A/R VS. F TABLE FOR THIS CASE IS THE SAME AS THE PRECEDING CASE

OUTPUT
BLOCK LOAD NO. 1

FINAL CYCLE NUMBER	UF	.40000000+05
CYCLIC STRESS RANGE	DP	.41450000+05
CYCLIC STRESS RATIO	R	.10000000-01
STEP SIZE	H	.10000000+01
PRECISION	PRC	.70000000+01
STEP CHANGE	ISC	0
START CYCLE PRINT	START	.40000000+05

N	A	B	DA/DN	DB/DN	DK(AT)	DK(B)
.00000000	.63000000-01	.12850000+00	.26711218-04	.18097792-04	.17713266+05	.12511296+05
.11150000+04	.11940182+00	.16363161+00	.19179390-03	.60082526-04	.29076634+05	.18398228+05
						.17522716+01